

# Medical Image Segmentation Employing Information Gain and Fuzzy C-Means Algorithm

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**Abstract**—In this paper, we proposed a new approach for image clustering to address the adverse effects of noise presented in the images. In particular, the concept of information gain has been incorporated into classical fuzzy c-means (FCM) algorithm in order to develop a robust clustering method. FCM is associated with high sensitivity to noise and produces non-homogenous clustering. To induce robustness to noise, the new clustering technique updates fuzzy membership values and cluster centroids based on information gain. The proposed method produces more homogeneous clustering and its performance can be verified at noisy and noise free images. Experiments have been performed on synthetic, CT liver images and compared with those of classical FCM and one of its robust variants. Moreover, the proposed algorithm has been validated on a data set of 30 carotid artery ultrasound images. Visual inspection of segmented images and clustering quality measures confirm that the proposed approach outperforms other clustering algorithms in comparison. Quantitative measures, in terms of PC and CE, also lead to similar conclusion. Hence, the proposed algorithm is robust to noise and produces homogenous clustering.

**Keywords**— FCM; Information Gain; Medical Image Clustering, Entropy.

## I. INTRODUCTION

Disease diagnosis based on medical imaging is an invaluable tool for medical experts to plan a patient rehabilitation process. Some of the contemporary medical imaging modalities being used by medical practitioners are ultrasound imagery, magnetic resonance imaging (MRI), computed tomography (CT) and digital mammography. Imaging based disease diagnostic helps medical experts, accurately diagnose the disease in a non-invasive way. As the volume of medical images is increasing day by day, it is no more feasible to analyze and interpret these images manually. A computer aided diagnostic (CAD) system is highly desirable to provide additional support to the radiologists. Particularly, computer algorithms are required to illustrate the structures and region of interest (ROI) automatically to the medical experts. These algorithms often segment a medical image into its constituent structures/regions in order to diagnose a disease. So, image segmentation algorithms play significant role in various medical image analysis tasks such as quantification of tissue volume [1], diagnosis [2], anatomical structure study [3], treatment planning [4] and computer aided surgery [5].

Several methods have been proposed in the literature to segment medical images [6-8]. Each has its own benefits and limitations but no single segmentation technique is able to address all the problems simultaneously.

Fuzzy c-means (FCM) clustering is one of the several existing segmentation techniques which take advantage of partial volume effect in images and especially in medical images. It is an unsupervised technique and has been successfully applied to feature analysis, clustering and classifier design in fields such as, medical imaging, geology, astronomy, image segmentation and target recognition [9-13]. FCM can classify the given image into specified number of clusters by grouping similar data points. Clusters are obtained iteratively by minimizing the cost function that depends upon the distance of pixels to the cluster centers. FCM is the generalization of k-means clustering which uses hard partitioning of data. Standard k-means is a popular technique in pattern recognition [14] that used hard partitioning of data. While in FCM, each data point may belong to more than one clusters with certain degrees of membership [15]. FCM is especially useful in medical image segmentation where objects in the images may not have well separated boundaries. In such situations, fuzzy based techniques are the suitable candidate approaches.

In this paper, we have extended the classical FCM clustering method by incorporating the concepts of information gain. The proposed scheme is named as Information Gain based Fuzzy C-Means (IGFCM) clustering. IGFCM is more robust to noise and produces more homogeneous clustering. Synthetic and CT liver images corrupted with different levels of noise are segmented using IGFCM and experimental results, based on visual inspection and clustering validity measures, validate the usefulness of the proposed approach. In addition to this, a dataset of 30 real carotid artery ultrasound images have been segmented by the proposed approach.

Rest of the paper is organized as follows. Section II describes the proposed scheme. Experimental results and discussion are presented in Section III. Finally, the research is concluded and future directions are set in Section IV.

## II. PROPOSED SCHEME

As described earlier, the proposed approach is an extension of classical FCM algorithm. So, in this section, we first describe the FCM algorithm and the mathematical

formulations involved and then describe the proposed IGFCM algorithm in detail.

#### A. Fuzzy C-means Clustering

Fuzzy c-means clustering is one of the most commonly used segmentation algorithm. It differs from standard k-means clustering in a sense that it integrates fuzzy information into the clustering process. It assigns pixels to clusters based on their fuzzy membership to a particular cluster. FCM strives to minimize the following cost function:

$$J = \sum_{j=1}^N \sum_{i=1}^C u_{ij}^m \|x_j - v_i\|^2 \quad (1)$$

Where  $u_{ij}$  shows the membership of pixel  $x_j$  to the  $i$ th cluster  $\forall x_j \in \Omega$  where  $\Omega$  represents the set of points that an image is composed of,  $C$  is total number of clusters,  $v_i$  is  $i$ th cluster centroid,  $\|\cdot\|$  is norm metric and  $m$  is the constant also known as fuzzy index or degree of fuzziness and is usually set to 2. When  $m$  set to zero, FCM becomes standard k-means clustering.

The cost function is minimized iteratively by updating clusters centroid. The following mathematical expressions are used to update the fuzzy membership functions and cluster centers:

$$u_{ij} = \frac{1}{\sum_{k=1}^c \left( \frac{\|x_j - v_i\|}{\|x_j - v_k\|} \right)^{\frac{2}{m-1}}} \quad (2)$$

$$v_i = \frac{\sum_{j=1}^N u_{ij} x_j}{\sum_{j=1}^N u_{ij}^m} \quad (3)$$

#### B. The Proposed Information Gain Based FCM

Classical FCM algorithm is highly sensitive to noise and may produces non-homogeneous clustering. To tackle the problem of noise effectively, the basic FCM framework is extended by exploiting the concepts of information gain. The idea of entropy was coined by Claude Shannon in his pioneering work on information theory [16]. It is the measure of uncertainty of a random variable and information gain is the measure of goodness of an attribute. In our proposed IGFCM algorithm, the input is the original image segmented by classical FCM technique but limiting it to some iterations only. For each pixel in the input image, we have computed entropy and information gain of its surrounding pixels in a certain neighborhood (5x5 in our case). The window size is set empirically and using 5x5 window size the proposed algorithm outperformed the other techniques. The proposed

technique then updates the fuzzy membership values of a pixel by swapping them together. The swapping order of the pixel fuzzy membership values is determined by the rank of its information gain values calculated for different clusters. The FCM iteration proceeds with the new membership values and newly computed centroids. This process continues until the difference between cluster centroid values in consecutive iterations reduces below than a particular threshold or maximum numbers of iterations have been invoked. This process is described in a step by step algorithm as following.

*The Proposed Algorithm:*

*Phase-I*

- Perform the FCM segmentation on input image.
  - Compute  $v_{kn}$  (initial cluster centroids) for all  $(k, n)$
  - Compute  $u_{jk}$  (initial fuzzy membership values) for all  $(j, k)$

Phase-I concludes after the image is segmented by 3 iterations of FCM algorithm. The output of this phase is fed to Phase-II for further computation.

*Phase-II*

1. Use a 5x5 window and iterate it over FCM segmented image.
2. Calculate the probability of each class.

$$p_i = n_i / N \quad \forall i \in \{1, C\}$$

where  $n_i$  represents total number of pixels in the window belonging to class  $i$ .  $N$  is total number of pixels in the mentioned window size.

3. Calculate the Entropy of each class using following equation

$$entropy(i) = -\sum_i p_i \log_2(p_i), \quad \forall i \in \{1, C\}$$

4. Calculate the between classes entropy for each two-class combination using expression below.

$$entropy(i, j) = -\sum_i p_i \log_2(p_i) - \sum_j p_j \log_2(p_j), \quad \forall (i, j) \in \{1, C\}$$

5. Calculate Expected Information (EI) using following expression.

$$EI = \sum_{i \in \{1, C\}} \sum_{j \in \{1, C\}} (p_i + p_j) entropy(i, j), \quad \text{where } i \neq j \text{ and } j > i$$

6. Calculate Information Gain for each class by following equation.

$$IG_i = entropy(i) - EI, \quad \forall i \in \{1, C\}$$

7. Compute  $u_{jk}^{sorted} = sort(u_{jk}, 'desc')$

8. Update  $u_{jk}^{New}$  using following expression

$$u_{jk}^{New} = u_{rank(IG, IG_j), k}^{sorted} \quad \forall (j, k) \in \{1, C\} \times \Omega$$

where  $IG$  is set of information gain values for all classes and  $rank(IG, i)$  returns the rank of number 'i' within a set of numbers  $IG$ .

9. Update  $v_{kn}$  from Eq. (3) using  $u_{jk}^{New}$  for each pixel  $n$ .
10. Set  $u_{jk} = u_{jk}^{New}$ .
11. Use  $u_{jk}$  and  $v_{kn}$  to perform FCM iteration.
12. Repeat all the above steps until the stopping criterion is met.

Repeat all the above steps until the stopping criterion is met. The stopping criterion is defined as, when either certain numbers of iterations of Phase-II are completed or the maximum difference between cluster centers at two successive iterations becomes less than a certain threshold. The defuzzification of  $u_{jk}$  yields the final image segmented by IGFCM algorithm. An illustration of above algorithmic steps is shown by an example in Appendix-I.

### C. Clustering Quality Measures

To evaluate the clustering performance for FCM, FLICM, and the proposed IGFCM algorithms, we have employed the partitioning coefficient (PC) and classification entropy (CE). High PC value indicates the better clustering while better clustering is expected at low value of CE [17-18].

## III. EXPERIMENTAL RESULTS AND DISCUSSION

In order to evaluate the performance, the proposed IGFCM algorithm has been applied to different images with various noise intensities. The images include synthetic and CT liver images. The proposed IGFCM algorithm has been compared to FCM and FLICM, both visually and quantitatively. We have used partition coefficient (PC) and classification entropy (CE) as clustering quality measures.

We first present the results of applying FCM, FLICM and IGFCM to a synthetic image of three segments, as shown in Figure 1 (a). To evaluate the segmentation performance of FCM, FLICM and IGFCM on this image, Gaussian noise of different variances (0.01, 0.02 and 0.03) has been added to the image to obtain 3 noisy versions of the original image. The performance of the aforesaid algorithms has been observed by applying them to the obtained noisy images. The quantitative results of segmentation for all noise levels are presented in the quantitative analysis subsection. However, in order to visually inspect the results, the noisy and segmented images are shown for noise variance 0.01 in Figure 1. As shown in Figure 1(e), IGFCM algorithm show the significance improvement to segment noisy image as compared to FCM and FLICM algorithms. Clear superiority of IGFCM can be observed over both FCM and FLICM as the image segmented by FCM and FLICM still contain a lot of noisy patterns while IGFCM produces much homogeneous segmentation.

The proposed algorithm is also applied to segment a CT liver image. Medical practitioners use details in liver CT images to diagnose various diseases. The CT liver image

(extracted region of interest), with tumor, is considered in our work. To check the robustness of the proposed IGFCM algorithm, Gaussian noise of variance 0.01 has been added to original image. Then, standard FCM, FLICM, and the proposed IGFCM algorithms has been applied to segment noisy image. There are three main segments in the liver CT image: normal liver tissue, tumorous liver tissues and the background area. The goal is to segregate these three segments in order to identify the tumorous liver tissue.

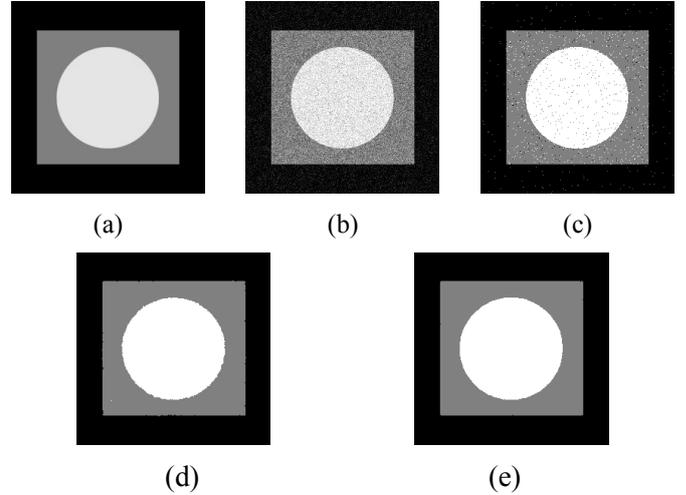


Figure 1. (a) The Synthetic image with 3 clusters (b) Noisy Synthetic image (noise variance 0.01), Images segmented by (c) FCM (d) FLICM (e) IGFCM

Figure 2 (a, b, c and d) shows original CT liver image and the resultant segmented images after applying the FCM and FLICM and the proposed IGFCM algorithms on the original image. Over-segmentation is immediately visible in case of both FCM and FLICM techniques. On the other hand, IGFCM produces more homogeneous clustering. The quantitative results shown in Table 1 also verify the superiority of the proposed IGFCM.

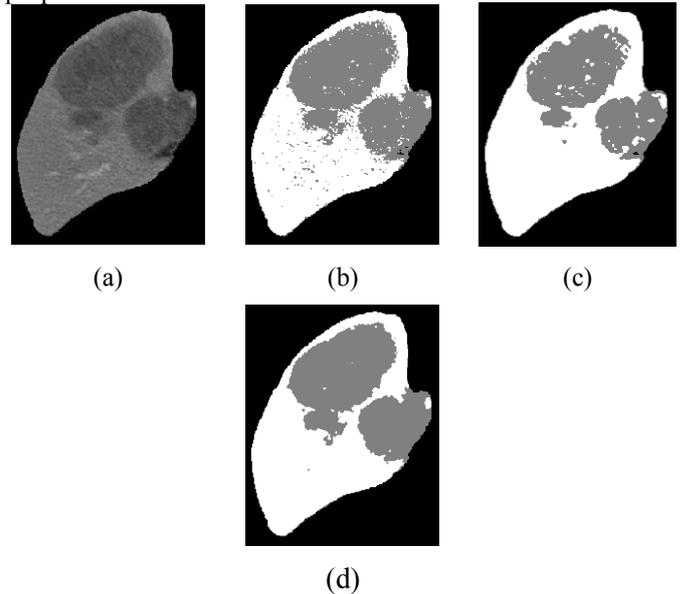


Figure 2. (a) Original CT liver Image, images segmented by (b) FCM, (c) FLICM (d) IGFCM

### A. Application to carotid artery ultrasound images segmentation

In order to assess the reliability of the proposed IGFCM algorithm as a general-purpose segmentation technique, a data set of 30 carotid artery ultrasound images have also been segmented using the proposed algorithm. The dataset has been obtained from Shifa International Hospital Islamabad, Pakistan. The carotid artery ultrasound images contain mainly three regions, namely the arterial wall, area inside the artery and background tissues. The objective of segmenting these images is to delineate each tissue separately. Figure 3 comprises of a sample carotid artery ultrasound image from the said dataset which is segmented by FCM, FLICM and the proposed IGFCM technique. It can be observed visually that the proposed algorithm effectively segregates the three regions of the image. Whereas, the images segmented by FCM and FLICM contain misclassified patterns in different regions (especially see the highlighted region in Figure 3). The advantage of the proposed IGFCM technique on carotid artery data set can also be observed by average quantitative clustering quality measures shown in Table-I.

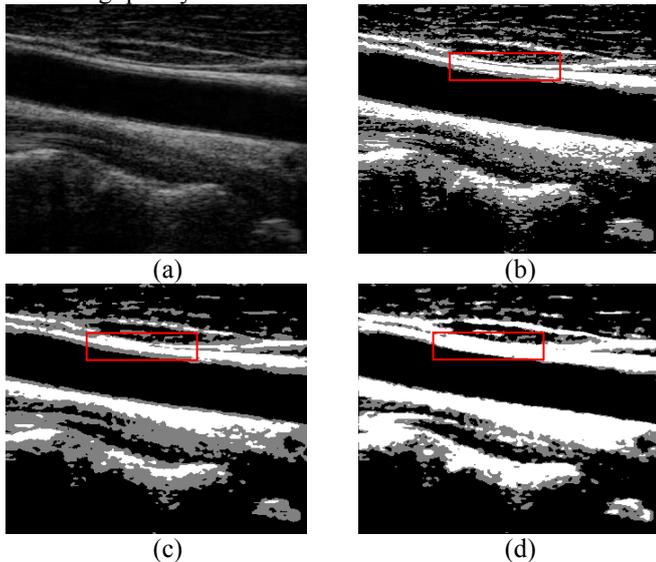


Figure 2. (a) Original carotid artery ultrasound image, images segmented by (b) FCM, (c) FLICM (d) IGFCM algorithm

### B. Quantitative Analysis

The clustering quality measures, Partitioning Coefficient (PC) and Classification Entropy (CE) has been computed for each of the segmented image. Table-I compare the PC and CE clustering quality measures. These measures are computed for the proposed IGFCM, FCM and FLICM algorithms for synthetic and liver CT images of different intensity of noise. A higher value of PC designates good segmentation capability while a lower value of CE is desirable. Both PC and CE vary between 0 and 1. The quantitative results indicate better segmentation using IGFCM than FCM and FLICM, as indicated by boldface values. The segmentation is not applicable to original synthetic image. However, the improved segmentation is evident from quantitative measures obtained

from segmentation of original liver CT image. The improvement in overall segmentation quality is even more significant when the input images contain noise of different variances. The results in almost all cases represent that IGFCM significantly outperform FCM and FLICM algorithms.

Finally, Table-I presents average PC and CE measures over 30 carotid artery ultrasound images segmented by using FCM, FLICM and the proposed IGFCM algorithm. The results have been presented for segmentation of both noisy and noise-free carotid artery ultrasound images. Like the synthetic and CT liver image, significantly improved quantitative measures have been obtained for carotid artery ultrasound images dataset, especially at higher noise levels. Therefore, the proposed technique can successfully be used to delineate different regions of carotid artery ultrasound images.

TABLE I. PC AND CE MEASURES FOR FCM, FLICM, AND IGFCM TECHNIQUES

	PC Measure				CE Measure			
	Original	var .01	var .02	var .03	Original	var .01	var .02	var .03
<b>Synthetic Image</b>								
FCM	-	0.9667	0.9659	0.9657	-	0.0702	0.0714	0.0721
FLICM	--	0.8866	0.8446	0.8107	--	0.2485	0.3237	0.3811
<b>IGFCM</b>	-	<b>0.9915</b>	<b>0.9914</b>	<b>0.9912</b>	-	<b>0.0138</b>	<b>0.0142</b>	<b>0.0145</b>
<b>CT Liver Image</b>								
FCM	0.9251	0.8300	0.8100	0.8000	0.1370	0.3000	0.3100	0.3200
FLICM	0.9417	0.7384	0.6625	0.6150	0.1015	0.4843	0.6037	0.6767
<b>IGFCM</b>	<b>0.9417</b>	<b>0.9123</b>	<b>0.9013</b>	<b>0.8910</b>	<b>0.1015</b>	<b>0.1401</b>	<b>0.1500</b>	<b>0.1633</b>
<b>Average results of 30 real carotid artery ultrasound images</b>								
FCM	0.7850	0.7735	0.7509	0.7297	0.3719	0.3886	0.4109	0.4373
FLICM	0.8000	0.7566	0.7141	0.6803	0.3833	0.4580	0.5267	0.5799
<b>IGFCM</b>	<b>0.8873</b>	<b>0.8497</b>	<b>0.8364</b>	<b>0.8145</b>	<b>0.1925</b>	<b>0.2472</b>	<b>0.2633</b>	<b>0.2686</b>

## IV. CONCLUSION

In this paper, a new clustering algorithm called Information Gain based Fuzzy C-Means (IGFCM) has been proposed. The conventional fuzzy c-means (FCM) algorithm is associated with high noise susceptibility and produces non homogeneous clustering. The proposed algorithm incorporates the concept of information gain to overcome the shortcomings of the FCM algorithm. It updates the fuzzy membership values of a pixel based on information gain computed from pixels in its local neighborhood. We have applied the proposed IGFCM algorithm to synthetic and CT Liver images and compared its performance with conventional FCM and FLICM algorithms. Experimental results indicate that IGFCM produces better clustering both visually and quantitatively. Different noisy versions of the said images have also been segmented successfully by the IGFCM algorithm compared to FCM and FLICM. Finally, a dataset of 30 noisy and noise-free carotid artery ultrasound images have been segmented by the proposed technique. Visual and quantitatively, the robustness

and effectiveness of the proposed algorithm can be verified as compared to FCM and FLICM techniques. Therefore, the proposed algorithm can be considered a reliable and robust image segmentation technique. In future, we intend to apply the proposed technique in feature space in associated with computational intelligence techniques can expect better segmentation performance.

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## APPENDIX-I

**Illustration of the algorithm:** An example is presented in this section which demonstrates a complete iteration of Phase-II of the proposed IGFCM algorithm. Phase-I of the algorithm is not discussed because it deals only simply with the application of classical FCM up to 3 iterations which is a well-studied problem.

In Phase-II, We have considered a local neighborhood of size 5x5 from the (FCM segmented) input image and demonstrated all the steps in Phase-II of the proposed algorithm. The local neighborhood shown is just one of several neighborhoods of the input image and the application of algorithm to other neighborhoods is straight forward. We now describe the step by step execution of Phase-II of the algorithm on given sample neighborhood.

## Step 1.

A sample local neighborhood is shown in Figure A1 where the pixel under consideration is shaded.

1	1	2	3	1
2	3	1	3	2
2	1	1	2	1
2	3	1	1	3
3	3	3	3	3

**Figure A1.** A sample pixel and corresponding 5x5 local neighborhoods

The given input fuzzy membership functions in this example are as following.

$$u_{jk} = [0.1 \ 0.7 \ 0.2]$$

## Step 2.

$$p_1 = \frac{8}{25}, p_2 = \frac{7}{25}, p_3 = \frac{10}{25}$$

## Step 3.

$$\text{entropy}(1) = -\frac{8}{25} \times \log_2\left(\frac{8}{25}\right) - \left(\frac{17}{25}\right) \times \log_2\left(\frac{17}{25}\right) = 0.9381$$

$$\text{entropy}(2) = 0.8354, \quad \text{entropy}(3) = 0.9972$$

## Step 4.

$$\text{entropy}(1,2) = -\frac{8}{25} \times \log_2\left(\frac{8}{25}\right) - \frac{7}{25} \times \log_2\left(\frac{7}{25}\right) = 1.0202$$

$$\text{entropy}(2,3) = 1.0229, \quad \text{entropy}(1,3) = 1.0548$$

## Step 5.

$$EI = \frac{15}{25} \times \text{entropy}(1,2) + \frac{17}{25} \times \text{entropy}(2,3) + \frac{18}{25} \times \text{entropy}(1,3) = 1.9845$$

## Step 6.

$$IG_1 = \text{entropy}(1) - EI = 0.9381 - 1.9845 = -1.0473$$

$$IG_2 = -1.1500, \quad IG_3 = -0.9882$$

## Step 7.

$$u_{jk}^{\text{sorted}} = [0.7, 0.2, 0.1]$$

## Step 8.

$$IG = [IG_1, IG_2, IG_3] = [-0.9882, -1.0473, -1.1500]$$

$$\text{rank}(IG, IG_1) = 2, \quad \text{rank}(IG, IG_2) = 3, \quad \text{rank}(IG, IG_3)$$

$$u_{1k}^{\text{New}} = u_{2k}^{\text{sorted}}, \quad u_{2k} = u_{3k}^{\text{sorted}}, \quad u_{3k} = u_{1k}^{\text{sorted}}$$

$$\text{Therefore, } u_{jk}^{\text{New}} = [0.2, 0.1, 0.7]$$

Until now, we have obtained the new fuzzy membership functions. We can use these new membership values to continue the process for further calculations (e.g. to find out new centroids as in step 9 of Phase-II).