Backstepping control design for two-wheeled self balancing robot

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***Abstract*—This paper presents different control schemes for tracking the pitch and yaw angle of a two-wheeled robot (TWR). Mathematical model of the TWR is developed using Lagrangian and Kirchhoff's current and voltage laws. The nonlinear system of TWR is first linearized at equilibrium point and then decomposed into two subsystems, a fourth and a second order system. The novelty discussed in this paper is the transformation of the fourth order system into strictly feedback form using input output linearization and is used to control the body pitch angle. The second order system is used to control the body yaw angle. The control strategies consisting of linear quadratic regulator (LQR) and iterative backstepping are developed to track the desired response. The simulation results ensure better performance for backstepping scheme as compared to the traditional LQR.**

***Index Terms*—Nonlinear control, Two-wheeled robot, Backstepping control, Euler-Lagrange, Feedback Linearization, Mathematical Modeling**

I. Introduction

Control of mobile robots is the main focus of researchers in recent years due to complex dynamics and highly nonlinearity. Robots are playing a key role in progressing industries. It is ubiquitous in different fields and facilitating the human in different applications. Nowadays, it is being used in daily life, such as in home, office, school and hospitals as guard robot, care robots, service robot and even as entertainment robots. A two-wheeled robot(TWR) is just like an inverted pendulum with two wheels, but its dynamics are different from the inverted pendulum due to its differentially driven wheels. The problem of stabilizing a system is challenging task when there are complex and nonlinear dynamics [1]. Because there is no single control scheme that outperforms in every scenario, there have been distributed efforts in controlling TWR such as zero moment point (ZMP) [2], pole placement [3] and LQR [4]–[7]. In [8], a neural network based controller is designed for better tracking with minimum control effort, of TWR. In [9], a sliding mode controller (SMC) is designed to balance a TWR. However, it does not track the desired reference input with better performance.

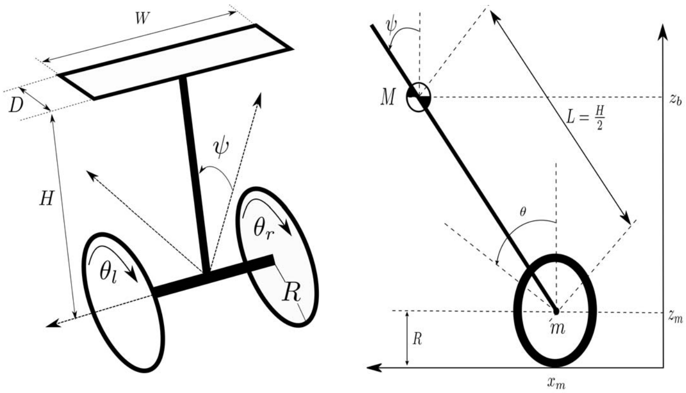
The pole placement controller [10] is designed to decrease settling time and steady state error. The controller is not robust and can be redesigned expediently and repetitiously till the desired results are achieved. In [11], Proportional derivative based fuzzy controller is designed to tackle the disturbances and parametric uncertainties. H_∞ controller based on partial feedback linearization using zero dynamics is designed in [12] which guaranteed the robustness and makes the robot move to the desired position with high speed. This controller control the revolving and curving motion of the robot. For the uncertainties which appear in the plant model, H_2 robust controller is designed and compared with the simulation and experimental results of linear quadratic regulator (LQR) controller. The comparison show that the H_2 controller is more effective than LQR controller [13].

A nonlinear controller using Lyapunov function has been formulated in [14] and [15] to enhance the stability of the proposed robot. The performance of the uncontrolled and SMC are analyzed in terms of position, speed and angle. The SMC shows better performance and provide higher level of disturbances reduction as compared to uncontrolled [16]. SMC is designed in [17] which deals with the modeling uncertainties and external disturbances, but introduce an effect of chattering. Second order SMC is used [18] to avoid chattering. Here the time derivative of the control input is considered because the use of the control input results in chattering. The robustness against the modeling errors has been improved by [19] presenting a backstepping with a sliding mode controller. Sliding mode controller is designed for stabilizing the mobile robot on the slopes by restricting the spinning motion in 3-degree of freedom motion dynamic equation [20]. μ-synthesis controller is designed in [21] guarantees the simulation and experimental results as compared to other controllers. Motivated by the above discussions, first we transformed the nonlinear system of TWR into linear form by using feedback linearization. Further we applied LQR and backstepping control schemes to attain the desired response.

This paper is structure as follows. In Section II, mathematical model of the addressed system is presented. LQR and backstepping control schemes for TWR are designed in Section III and IV, respectively. Simulation results are discussed in Section V. Finally, the conclusions are drawn in Section VI.

II. System Modeling

Fig.1 shows the structure model of TWR. The robot has a gyro-sensor and optical encoder attached toboth wheel motors. Rotational angle (pitch) of the body and rotational angle of the wheels are available measurement outputs.



(a) System Mathematical Model (b) Side view

Fig. 1: TWR

where ψ is the rotational angle of the body, θ is the rotational angle of the wheels, (x_b, z_b) is the position of body center mass and (x_m, z_m) is the position of the wheel center in x and z axis, R_w is the radius of the wheels, and L is the distance between the wheel centers axel to the body center mass. Table I shows the system parameters with their values used in modelling of the system. To obtain mathematical model of the system a conventional model procedure is adoped. Using Lagarangian mechanics the kinetics and potential energies are as follows.

TABLE I: Parameters of TWR

Parameters	**Symbols**	**Unit**
Gravity Acceleration	g = 9.8	ms^{-2}
Mass of Wheel	m =0.03	Kg
Radius of Wheel	R = 0.04	m
Inertia Moment of the Wheel	J_w = $mR^2/2$	Kgm^2
Intermediate Body Mass	M = 0.6	Kg
Intermediate Body Width	W = 0.14	m
Intermediate Body Depth	D = 0.04	m
Intermediate Body Height	H = 0.144	m
Center of Mass from Wheel Axel	$L = H/2$	m
Pitch Inertial Mass	J_ψ = $ML^2/3$	Kgm^2
Yaw Inertial Mass	J_ϕ = $M(W^2 + D^2)/2$	Kgm^2
Motor Inerital Mass	J_m = 1 × 10^{-5}	Kgm^2
Armature Resistance	R_m = 6.69	Ω
Back EMF of Motor	K_b = 0.468	$V sec/rad$
Motor Torque Constt	K_t = 0.317	Nm/A
Gear Ratio	n = 1	
Friction between Body & DC Motor	f_m = 0.0022	
Friction between Wheel & Floor	f_w = 0.0012	

$$T_1 = \frac{1}{2}m(\dot{x}_l^2 + \dot{y}_l^2 + \dot{z}_l^2 + \dot{x}_r^2 + \dot{y}_r^2 + \dot{z}_r^2) + \frac{1}{2}M(\dot{x}_b^2 + \dot{y}_b^2 + \dot{z}_b^2) \tag{1}$$

$$T_2 = \frac{1}{2}(J_w\dot{\theta}_l^2 + J_w\dot{\theta}_r^2 + J_\psi\dot{\psi}^2 + J_\phi\dot{\phi}^2 + n^2J_m(\dot{\theta}_l - \dot{\psi})^2 + n^2J_m(\dot{\theta}_r - \dot{\psi})^2) \tag{2}$$

$$U = mgz_l + mgz_r + Mgz_b \tag{3}$$

where T_1 is the translational kinetic energy, T_2 is the rotational kinetic energy, and U is the gravitaitonal potential energy of the plant. Euler-Lagrangian equations for the addresed systems are as follows.

$$Z = T_1 + T_2 - U \tag{4}$$

$$\frac{d}{dt}(\frac{\partial Z}{\partial \dot{\theta}}) - \frac{\partial Z}{\partial \theta} = F_\theta \tag{5}$$

$$\frac{d}{dt}(\frac{\partial Z}{\partial \dot{\psi}}) - \frac{\partial Z}{\partial \psi} = F_\psi \tag{6}$$

$$\frac{d}{dt}(\frac{\partial Z}{\partial \dot{\phi}}) - \frac{\partial Z}{\partial \phi} = F_\phi \tag{7}$$

The linearized model of the robot can be obtained by linearizing Lagrange equations (5), (6), and (7) at the neighbourhood of standing posture. We assumed that the pitch ψ is small enough and approximate $\sin(\psi) \to \psi$, $\cos(\psi) \to 1$, and the squared terms like ψ^2 are neglected. As a result, the following equations are obtained.

$$F_\theta = ((2m + M)R_w^2 + 2J_w + 2n^2J_m)\ddot{\theta} + (MR_wL - 2n^2J_m)\ddot{\psi} \tag{8}$$

$$F_\psi = (MLR_w - 2n^2J_m)\ddot{\theta} + (ML^2 + J_\psi + 2n^2J_m)\ddot{\psi} - MgL\psi \tag{9}$$

$$F_\phi = \frac{1}{2}mW^2 + J_\phi + \frac{W}{2R_w^2}(J_w + n^2J_m)\ddot{\phi} \tag{10}$$

The desired balancing of the TWR is largely dependent on a pair of DC motors. The velocity of both wheels plus direction of TWR are controlled by varying the speed of each motor. Considering the torque of the DC motor and viscous friction, generalized forces F_θ, F_ψ and F_ϕ are given as follows.

$$F_\theta = \alpha(u_l + u_r) + 2\beta\dot{\psi} - 2(\beta + f_w)\dot{\theta} \tag{11}$$

$$F_\psi = -\alpha(u_l + u_r) - 2\beta\dot{\psi} + 2\beta\dot{\theta} \tag{12}$$

$$F_\phi = -\frac{W}{2R_w}\alpha(u_r - u_l) - \frac{W^2}{2R_w^2}(\beta + f_w)\dot{\phi} \tag{13}$$

where $\alpha = \frac{nK_t}{R_m}$, $\beta = \frac{nK_tK_b}{R_m} + f_m$ and i is the current through DC motor. DC motor equation for left wheel is

$$L_m i_l = u_l + K_b(\dot{\psi} - \dot{\theta}_l) - R_m i_l \tag{14}$$

DC motor equation for right wheel is

$$L_m i_r = u_r + K_b(\dot{\psi} - \dot{\theta}_l) - R_m i_r \tag{15}$$

where u_l and u_r are the voltages to the left and right DC motors respectively. Since the inductance of DC motor L_m is very small, so neglecting this can lead us to the following equations.

$$i_l = \frac{u_l + K_b(\dot{\psi} - \dot{\theta}_l)}{R_m} \tag{16}$$

$$i_r = \frac{u_r + K_b(\dot{\psi} - \dot{\theta}_r)}{R_m} \tag{17}$$

After substituting in equations (8), (9) and (10) the system equations in-terms of input voltages are as follows

$$((2m + M)R^2 + 2J_w + 2n^2 J_m)\ddot{\theta} + (MLR - 2n^2 J_m)\ddot{\psi} + 2(\beta + f_w)\dot{\theta} - 2B\dot{\psi} = \alpha u_1 \tag{18}$$

$$(MLR - 2n^2 J_m)\ddot{\theta} + (ML^2 + J_\psi + 2n^2 J_m)\ddot{\psi} - 2\beta\dot{\theta} + 2\beta\psi = -\alpha u_1 \tag{19}$$

$$(\frac{1}{2}mW^2 + \frac{W^2}{2R^2}(J_w + n^2 J_m) + J_\phi)\ddot{\phi} + \frac{W^2}{2R^2}(\beta + f_w)\dot{\phi} = \frac{W}{2R}\alpha u_2 \tag{20}$$

Let $\theta = x_1$, $\psi = x_2$, $\dot{\theta} = x_3$, $\dot{\psi} = x_4$, $\phi = x_5$, $\dot{\phi} = x_6$ the state-space equations are obtained and system is transformed into two subsystems i.e: fourth and second order system.

$$\dot{x}_{1a} = \begin{cases} x_3 \\ x_4 \\ -409.71x_2 - 162.12x_3 + 162.12x_4 + 175.57u_1 \\ 269.62x_2 + 78.14x_3 - 78.14x_4 - 57.95u_1 \end{cases} \tag{21}$$

$$\dot{x}_{1b} = \begin{cases} x_6 \\ -95.56x_6 + 53.07u_2 \end{cases} \tag{22}$$

where $\dot{x}_{1a} = [\dot{x}_1\ \dot{x}_2\ \dot{x}_3\ \dot{x}_4]$ and $\dot{x}_{1b} = [\dot{x}_5\ \dot{x}_6]$. As we know that backstepping technique can only be applied to system dynamics which are in normal feedback form, therefore, the fourth order system is converted into normal feedback form using feedback linearization. The transfromed system obtains after feedback linearization is as follows

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ \dot{z}_3 &= z_4 \\ \dot{z}_4 &= 11695.23z_2 + 269.62z_3 - 240.28z_4 - 5478.98u_1 \end{aligned} \tag{23}$$

III. Linear Quadratic Regulator

Linear Quadratic Regulator (LQR) is an optimal control technique in the absence of unwanted external disturbances and is usually the choice when complete information of the system is known [21]. LQR calculates the optimal gains in order to minimize the performance index, which results in the enhancement of the system's performance. It is desired to have maximum state regulation so that the system states are driven strongly to equilibrium, which requires high control effort. LQR provides a trade-off between state regulation and control effort. Consider the following generalized system

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

where $x \in R^4$ and $u \in R$ are the state and input variables respectively. The following performance index gives solution for the optimal control problem.

$$J(x,t) = \int_t^{t_f} (x^T Qx + u^T Ru) d\tau \tag{24}$$

However, for the nonlinear system it is not easy to solve the above performance index. Assuming infinite horizon and system to be controllable, the above performance index is reduced to algebraic Riccatti's equation.

$$A^T P + PA - PBR^{-1}B^T P = 0 \tag{25}$$

The value of feedback gain is computed by evaluating the Riccatti equation for different values of Q and R. Q matrix is assumed as

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{26}$$

Using (25), P matrix is calculated as

$$P = \begin{bmatrix} 2.8244 & 1.2104 & 0.0488 & 0.0002 \\ 1.2104 & 3.2647 & 0.2313 & 0.0009 \\ 0.0488 & 0.2313 & 0.0603 & 0.0002 \\ 0.0002 & 0.0009 & 0.0002 & 00000 \end{bmatrix} \tag{27}$$

The value of R is a scalar because there is a single control input. The feedback gain matrix is calculated as follows.

$$K = R^{-1}B^T P = [-750\ \ -67\ \ -2\ \ 0.005] \tag{28}$$

IV. Backstepping

Backstepping is a nonlinear control scheme proposed for a special class of linear and non-linear dynamical systems which are in the strictly feedback form. Here the system is divided into four subsystems. A virtual controller is designed for each subsystem in three steps, while the actual control law is designed in the last step [22].
Let the error variables are

$$e_1 = z_1 - z_d \tag{29}$$

$$e_2 = z_2 - \propto_1 \tag{30}$$

$$e_3 = z_3 - \propto_2 \tag{31}$$

$$e_4 = z_4 - \propto_3 \tag{32}$$

Where z_d is the desired position to be tracked and $\propto_1, \propto_2, \propto_3$ are the stabilizing functions for the subsystem in respective iteration.

$$\dot{e}_1 = \dot{z}_1 - \dot{z}_d = z_2 - \dot{z}_d$$

$$\dot{e}_1 = e_2 + \propto_1 - \dot{z}_d \tag{33}$$

$\propto_1$ should be selected through control lyapunov function to guarantee the stability of the system as:

$$V_1 = \frac{1}{2}{e_1}^2 \tag{34}$$

$$\dot{V}_1 = e_1\dot{e}_1 = e_1(e_2 + \propto_1 - \dot{z}_d)$$

$$\dot{V}_1 = e_1 e_2 + e_1(\propto_1 - \dot{z}_d)$$

$$\propto_1 = -c_1 e_1 + \dot{z}_d \tag{35}$$

Where $c_1 > 0$ is a design parameter. Equation (33) can be exponential stable as $t \to \infty$

$$\dot{V}_1 = -c_1{e_1}^2 + e_1 e_2 \tag{36}$$

$$\dot{e}_1 = -c_1 e_1 + e_2 \tag{37}$$

From equation (30), the state space equation for e_2 is:

$$\dot{e}_2 = z_2 - \dot{\propto}_1 = z_3 - \dot{\propto}_1$$

$$\dot{e}_2 = e_3 + \propto_2 - \dot{\propto}_1 \tag{38}$$

$$\dot{\propto}_1 = -c_1\dot{e}_1 + \ddot{z}_d = -c_1(-c_1 e_1 + e_2) + \ddot{z}_d \tag{39}$$

Since equation (38) includes the information of equation (33),the CLF is selected as:

$$V_2 = V_1 + \frac{1}{2}{e_2}^2 \tag{40}$$

$$\dot{V}_2 = \dot{V}_1 + e_2\dot{e}_2$$

$$\dot{V}_2 = -c_1{e_1}^2 + e_1 e_2 + e_2(e_3 + \propto_2 - \dot{\propto}_1)$$

$$\dot{V}_2 = -c_1{e_1}^2 + e_2 e_3 + e_2(e_1 + \propto_2 - \dot{\propto}_1)$$

$$\propto_2 = -c_2 e_2 - e_1 + \dot{\propto}_1 \tag{41}$$

By putting this equation into equation (38), we get

$$\dot{e}_2 = -e_1 - c_2 e_2 + e_3 \tag{42}$$

Where $c_2 > 0$ is a design parameter and $\propto_2$ can be rearranged as:

$$\propto_2 = -c_2 e_2 - e_1 - c_1(e_2 - c_1 e_1) + \ddot{z}_d$$

$$\propto_2 = -(c_1 + c_2)e_2 - (1 - {c_1}^2)e_1 + \ddot{z}_d \tag{43}$$

$$\dot{V}_2 = -c_1{e_1}^2 - c_2{e_2}^2 + e_2 e_3 \tag{44}$$

Since from equation (31), the state space equation for e_3 is:

$$\dot{e}_3 = \dot{z}_3 - \dot{\propto}_2 = z_4 - \dot{\propto}_2$$

$$\dot{e}_3 = e_4 + \propto_3 - \dot{\propto}_2 \tag{45}$$

From equation (41) and equation (39), we conclude $\dot{\propto}_2 = -c_2\dot{e}_2 - \dot{e}_1 + \ddot{\propto}_1$
where $\ddot{\propto}_1 = {c_1}^2\dot{e}_1 - c_1\dot{e}_2 + \dddot{z}_d$, we get

$$\dot{\propto}_2 = -(c_1 + c_2)\dot{e}_2 - (1 - {c_1}^2)\dot{e}_1 + \dddot{z}_d$$

$$\dot{\propto}_2 = (c_1 + c_2)(e_3 + \propto_2 - \dot{\propto}_1)(1 - {c_1}^2)(e_2 - c_1 e_1) + \dddot{z}_d$$

$$\begin{aligned}\dot{\propto}_2 =& (2c_1 - {c_1}^3 + c_2)e_1 - (1 - {c_1}^2 - {c_2}^2 - c_1 c_2)e_2 \\ & - (c_1 + c_2)e_3 + \dddot{z}_d\end{aligned} \tag{46}$$

Since equation (45) includes the information of equations (33) and (38), the CLF is selected as:

$$V_3 = V_2 + \frac{1}{2}{e_3}^2 \tag{47}$$

$$\dot{V}_3 = \dot{V}_2 + e_3\dot{e}_3$$

$$\dot{V}_3 = -c_1{e_1}^2 - c_2{e_2}^2 + e_2 e_3 + e_3(e_4 + \propto_3 - \dot{\propto}_2) \tag{48}$$

$$\begin{aligned}\dot{V}_3 =& - c_1{e_1}^2 - c_2{e_2}^2 + e_3 e_4 \\ & + e_3(e_2 + \propto_3 - \dot{\propto}_2)\end{aligned} \tag{49}$$

$$\propto_3 = -c_3 e_3 - e_2 + \dot{\propto}_2 \tag{50}$$

Where $c_3 > 0$ is a design parameter, by putting this equation into equation (45), we get

$$\dot{e}_3 = -e_2 - c_3 e_3 + e_4 \tag{51}$$

$$\begin{aligned}\propto_3 =& - c_3 e_3 - e_2 - (-2c_1 + {c_1}^3 - c_2)e_1 - (1 \\ & - {c_1}^2 - {c_2}^2 - c_1 c_2)e_2 - (c_1 + c_2)e_3 + \dddot{z}_d\end{aligned}$$

$$\begin{aligned}\propto_3 =& - (-2c_1 + {c_1}^3 - c_2)e_1 - (2 - {c_1}^2 - {c_2}^2 \\ & - c_1 c_2)e_2 - (c_1 + c_2 + c_3)e_3 + \dddot{z}_d\end{aligned} \tag{52}$$

$$\begin{aligned}\dot{\alpha}_3 =& - (c_1 + c_2 + c_3)(e_4 - c_3 e_3 - e_2) + (c_1 c_2 \\ & + {c_1}^2 + {c_2}^2 - 2)(e_3 - c_2 e_2 - e_1 + (2c_1 c_2 \\ & - {c_1}^3)(-c_1 e_1 + e_2) + \ddddot{z}_d)\end{aligned}$$

$$\dot{V}_3 = -c_1{e_1}^2 - c_2{e_2}^2 - c_3{e_3}^2 + e_3 e_4 \tag{53}$$

From equation (53), it is clear that $\dot{V}_3$ is not negative definite. Differentiating equation (53) we get

$$\dot{e}_4 = \dot{z}_4 - \dot{\alpha}_3 \tag{54}$$

Substituting the value of $\dot{z}_4$ in above equation

$$\begin{aligned}\dot{e}_4 =& 11695.2334 z_2 + 269.6273 z_3 - 240.2855 z_4 - 5478.988 u_1 \\ & - \dot{\alpha}_3\end{aligned} \tag{55}$$

The final Lyapunov candidate function can be written as

$$V_4 = V_3 + \frac{1}{2}{e_4}^2 \tag{56}$$

Putting equation (53) and (54) in equation (56) we get

$$\begin{aligned}\dot{V}_4 =& - c_1{e_1}^2 - {c_2}^2 e_2 - {c_3}^2 e_3 + e_3 e_4 + e_4(11695.23 z_2 \\ & + 269.62 z_3 - 240.28 z_4 - 5478.9 u_1 - \dot{\alpha}_3)\end{aligned} \tag{57}$$

Now the final control input is obtained as

$$\begin{aligned}u_1 =& \frac{1}{5478.9}\Big(c_4 e_4 + e_3 + 11695.23(e_2 + \alpha_1) + 269.62(e_3 \\ & + \alpha_2) - 240.28(e_4 + \alpha_3) - \dot{\alpha}_3)\Big)\end{aligned} \tag{58}$$

Using the control input u_1, $\dot{V}_4 < 0$ is negative definite, so the transformed system obtained after backstepping approach is asymptotically stable. Similarly for the system described in equation (22) the control input designed as

$$u_2 = \frac{1}{53.0787}(-z_5 + 95.5684\alpha_1(z_5) - c_6 z_6) \quad (59)$$

V. Simulation Results

Based on the feedback linearized model of TWR, both LQR and backstepping have been simulated for the proposed system. The model has been subjected to various desired/test inputs in order to analyze the dynamic characteristics of the proposed control scheme. For each reference input, the response that corresponds to both the control laws have been simulated in Matlab/Simulink. It is observed from Fig.(2) that output tracks the desired trajectory with a steady state error of 0.07 using LQR controller, when subjected to sinusoidal reference signal. Furthermore, the settling time is approximately 0.3 seconds. This implies that TWR takes 0.35 seconds to keep its intermediate body in upward direction.

Backstepping controller is also simulated in Matlab/Simulink. Fig.(3) shows that the actual trajectory of the body pitch successfully track the desired trajectory (sinusoid) when using backstepping controller. Fig.(4) shows the response of the actual trajectory of body yaw angle when subjected to ramp input, using backstepping controller. The steady state error for body pitch angle is 0.2 percent, and for body yaw angle is 0.1 percent. Settling time is 0.6 seconds for body pitch angle and 0.3 seconds for body yaw angle.

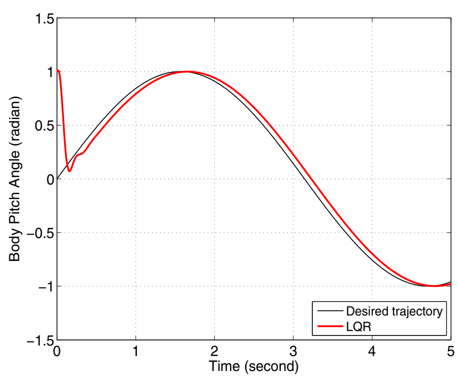


Fig. 2: Trajectory tracking of desired response using LQR

The motion of the TWR in X-Y plane is shown in Fig. 5. It is evident from simulation results that backstepping can track the desired input with better performance in terms of transient as well as steady state parameters than LQR.

VI. Conclusion

In this paper, mathematical model was obtained using Lagrangian approach. The mathematical model was then linearized at equilibrium point and decomposed into two subsystems, fourth and a second order system. The fourth order system was not straight forward input-state linearizable, because the system is not full state linearizable. Furthermore,

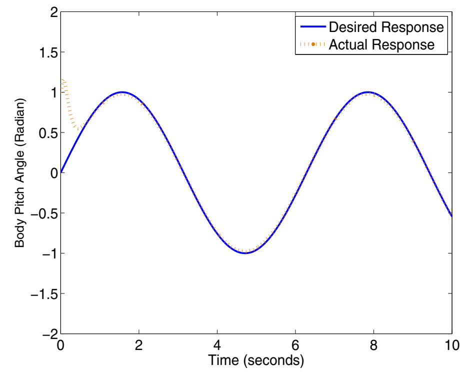


Fig. 3: Trajectory tracking of body pitch angle using Backstepping

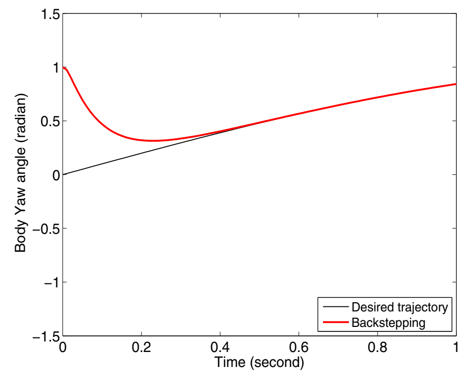


Fig. 4: Trajectory Tracking of body yaw angle using Backstepping

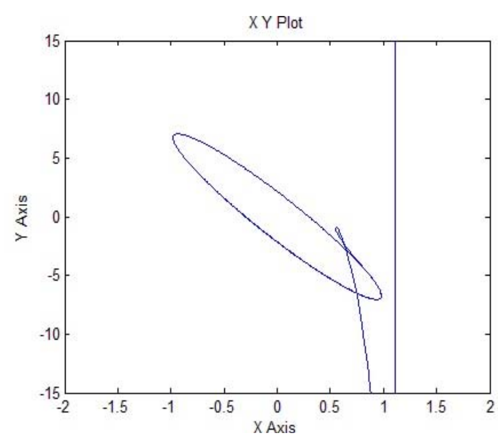


Fig. 5: Motion of TWR in X-Y Plane

the zero-dynamics of the system are also unstable. The relative degree of the system was improved by modifying the output of the system. Feedback gains for LQR controller were determined by evaluating Riccatti equation. Moreover, control law is designed using backstepping technique which has more freedom of attaining the stable response by changing gains at each step. The simulation results show that nonlinear backstepping controller's performance is better than LQR.

References

[1] M. Gasperi and P. Hurbain, *Extreme NXT: Extending the LEGO Mindstorms NXT to the next level*. Apress, 2010.

[2] D. Choi, M. Kim, and J.-H. Oh, "Development of a rapid mobile robot with a multi-degree-of-freedom inverted pendulum using the model-based zero-moment point stabilization method," *Advanced Robotics*, vol. 26, no. 5-6, pp. 515–535, 2012.

[3] F. Grasser, A. D'arrigo, S. Colombi, and A. C. Rufer, "Joe: a mobile, inverted pendulum," *IEEE Transactions on industrial electronics*, vol. 49, no. 1, pp. 107–114, 2002.

[4] S. Ahmad and M. O. Tokhi, "Linear quadratic regulator (lqr) approach for lifting and stabilizing of two-wheeled wheelchair," in *4th International Conference On Mechatronics*, pp. 1–6, 2011.

[5] D. R. Jones and K. A. Stol, "Modelling and stability control of two-wheeled robots in low-traction environments," in *Australasian Conference on Robotics and Automation, Brisbane, Australia*, 2010.

[6] S. Kalra, D. Patel, and K. Stol, "Design and hybrid control of a two wheeled robotic platform," in *Australasian Conference on Robotics and Automation*, 2007.

[7] C. Xu, M. Li, and F. Pan, "The system design and lqr control of a two-wheels self-balancing mobile robot," in *International Conference on Electrical and Control Engineering (ICECE)*, pp. 2786–2789, 2011.

[8] S. Jung and S. S. Kim, "Control experiment of a wheel-driven mobile inverted pendulum using neural network," *IEEE Transactions on Control Systems Technology*, vol. 16, no. 2, pp. 297–303, 2008.

[9] S. Nawawi, M. Ahmad, J. Osman, A. Husain, and M. Abdollah, "Controller design for two-wheels inverted pendulum mobile robot using pismc," in *4th Student Conference on Research and Development*, pp. 194–199, 2006.

[10] S. W. Nawawi, "Real-time control system for a two-wheeled inverted pendulum mobile robot," in *Advanced knowledge application in practice*, InTech, 2010.

[11] B. Gao, X. Zhang, W. Yao, Y. Guo, Y. Bao, and J. Xie, "Dynamics and motion control of an underactuated two-wheeled mobile robot," in *IEEE International Conference on Cyber Technology in Automation, Control, and Intelligent Systems*, pp. 179–184, 2012.

[12] A. Shimada and N. Hatakeyama, "Movement control of two-wheeled inverted pendulum robots considering robustness," in *SICE Annual Conference*, pp. 3361–3366, 2008.

[13] T. Kanada, Y. Watanabe, and G. Chen, "Robust h_2 control for two-wheeled inverted pendulum using lego mindstorms," in *Australian Control Conference (AUCC)*, pp. 136–141, IEEE, 2011.

[14] Z. Kausar, K. Stol, and N. Patel, "Nonlinear control design using lyapunov function for two-wheeled mobile robots," in *International Conference on Mechatronics and Machine Vision in Practice*, pp. 123–128, 2012.

[15] N. Ali, W. Alam, A. U. Rehman, and M. Pervaiz, "State and disturbance observer based control for a class of linear uncertain systems," in *International Conference on Frontiers of Information Technology*, pp. 139–143, 2017.

[16] N. A. Ghani, N. M. Yatim, and N. Azmi, "Comparative assessment for two wheels inverted pendulum mobile robot using robust control," in *International Conference on Control Automation and Systems (ICCAS)*, pp. 562–567, 2010.

[17] O. Khan, A. U. Rehman, and M. Pervaiz, "Beyond linear control approaches-sliding mode control of flexible robotic manipulator," in *International Conference on Frontiers of Information Technology*, pp. 1–6, 2016.

[18] H. Aithal and S. Janardhanan, "Trajectory tracking of two wheeled mobile robot using higher order sliding mode control," in *International Conference on Control Computing Communication & Materials (ICCCM)*, pp. 1–4, IEEE, 2013.

[19] C. Acar and T. Murakami, "Underactuated two-wheeled mobile manipulator control using nonlinear backstepping method," in *34th Annual Conference of IEEE Industrial Electronics*, pp. 1680–1685, 2008.

[20] M.-S. Key, C.-G. Jeon, and D. S. Yoo, "Sliding mode control for a two-wheeled inverted pendulum mobile robot driving on uniform slopes," in *International Conference on Control, Automation and Systems (ICCAS)*, pp. 2159–2162, IEEE, 2012.

[21] D.-W. Gu, P. H. Petkov, and M. M. Konstantinov, "Robust control of self-balancing two-wheeled robot," in *Robust Control Design with MATLAB®*, pp. 435–454, Springer, 2013.

[22] O. Khan, M. Pervaiz, E. Ahmad, and J. Iqbal, "On the derivation of novel model and sophisticated control of flexible joint manipulator," *Revue Roumaine des Sciences Techniques-Serie Electrotechnique et Energetique*, vol. 62, no. 1, pp. 103–108, 2017.