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Robust Fault Detection and Isolation for Uncertain Switched Systems under Asynchronous Switching with Adaptive Threshold ⋆

Muhammad Taskeen Raza * **Abdul Qayyum Khan** *
Ghulam Mustafa * **Muhammad Abid** *

* *Pakistan Institute of Engineering and Applied Sciences, Nilore, Islamabad, Pakistan*
(e-mail: [taskeenraza,aqkhan,gm,mabid]@pieas.edu.pk).

Abstract: The problem of fault detection and isolation for uncertain continuous-time linear switched systems in the presence of disturbances and noise is addressed in this paper. A robust residual generator is proposed which is based on asynchronously switching filters, where "asynchronous" means there is a lag between switching of filters and subsystems. To address the issue, fault detection problem is formulated as mixed H_-/H_∞ filtering problem. In proposed H_-/H_∞ technique, the effect of fault on residual signal is fixed to some maximum possible index and then influence of unknown inputs (disturbances and noise) on residual is minimized. In addition proposed filter has prominence of having fault isolation capability along with fault detection. To improve the fault detection capability adaptive threshold is set which takes into account local disturbance levels, the current operational mode and applied input signal. Then, solution is designed for boost converter switched system application and simulations are presented to illustrate the efficacy of the proposed framework.

Keywords: fault detection, switched systems, asynchronous switching, average dwell time

1. INTRODUCTION

In the last few years, due to their significance in theory and practical applications, switched systems have fascinated many researchers. These systems are significant in modeling complex dynamical processes into linear multiple sub-systems Abdo et al. (2013) and have numerous applications in control of robotics, mechanical systems, automotive industry, aircraft and air traffic control, switched power converters, and in many other fields, see for details Liberzon (2003). Switched systems are a class of hybrid systems consisting of subsystems, which have either continuous-time or discrete-time dynamics, and a switching signal which governs the activation of any particular subsystem along the trajectory of the switched system at any instant of time. Like other technical systems, abnormalities (faults) in sensors, actuators and process components of switched systems are inevitable. Along with stability issues, the occurrence of faults turns the situation rather difficult o handle. It is therefore, highly required to develop tools, schemes and methodologies of diagnosing faults for switched systems. Notice that fault detection and isolation (FDI) for dynamical systems is an active area of research, see for instance, Frank et al. (2000); Ding (2008); Chen and Patton (2012), wherein model-based FDI has drawn the attention of the researchers. The basic idea of model based fault detection is to generate a residual signal (symptom signal) by comparing the measured output signals of a physical system with the estimated outputs. Uncertainties are quite often present in system model; due to aging effect, change of process components, linearizion error, and inaccurate modeling of complex systems Ding (2008). To cope with this problem, the FDI system has to be maximally sensitive to the occurring faults in the system and at the same time maximally robust against unknown inputs. To this end, a range of optimization indices have been proposed. Few of those are, H_2/H_2, H_∞/H_∞, H_-/H_∞ Ding (2008), whereas the H_-/H_∞ index is of particular interest in our research, because of its twofold nature for the solution of the said problem. The H_- index takes into account the minimum influence of faults, while the H_∞ norm considers the worst-case effect of unknown inputs on the residual signal.

FDI problem for switched systems is being considered actively but has not been intensively studied so far. To mention few of them, fault detection (FD) problem for discrete time switched systems is considered by Belkhiat et al. (2011), while for continuous case Wang et al. (2010). Abdo et al. (2011) considered fault detection for uncertain discrete time switched systems. Wang et al. (2007) utilized H_-/H_∞ performance index for optimal fault detection in switched systems. However, all the aforementioned efforts for the problem have assumed that the fault detection filter is switching with the subsystems in synchronous manner. In this way, the problem is reduced to multiple linear dynamical systems simply. While in practice

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it takes time to identify the particular subsystem active at any instant of time. Therefore, the phenomenon of asynchronous switching between filter and system exists in general. In asynchronous switching there is a lag between filter and subsystem. Although the asynchronous problem has gained attention of FD research community in recent years Belkhiat et al. (2011); Du et al. (2013), still it deserves more attention to be paid. Belkhiat et al. (2011) studied fault detection problem for discrete switched systems with the assumption that the governing switching signal is unknown. Particularly, the work of Du et al. (2013), motivated us to explore further the asynchronous switching problem in fault diagnosis. In aforementioned work H_∞ filtering technique is used to formulate and design the filter structure.

In this paper, we present a solution to the fault detection and isolation problem for uncertain continuous-time switched systems under asynchronous switching case. This work is extension of our results Raza et al. (2014), by taking norm-bounded model uncertainties into account and instead of fixed threshold, which is conservative, proposing the adaptive threshold setting for this problem.

The rest of the paper is organized as follows: In Section 2, preliminaries and problem formulation are given. The solution to the problem is derived in Section 3. Section 4 is concerned with threshold computation and residual evaluation steps in fault detection. In Section 5, design example is given to show the effectiveness of the results, followed by conclusion in Section 6

2. PRELIMINARIES AND PROBLEM FORMULATION

2.1 Notations and Assumptions

The notations used in this paper are fairly common and standard. In this paper following assumptions are made: ADT switching constraint. The pair (A_i, C_i) is detectable. For (1), fault detectability condition, $C_i(sI - A_i)^{-1}B_{fij} + D_{fij} \neq 0$ holds, where B_{fij} and D_{fij} denote the jth columns of B_{fi} and D_{fi}, respectively and matrix D_{fi} is invertible which implies also that number of faults are equal to number of outputs, that is, $q = m$

2.2 Switched System and Fault Detection Filters Models - Asynchronous Switching Problem

Consider the following class of continuous-time switched systems

$$\begin{aligned} \dot{x}(t) &= \bar{A}_{\sigma(t)}x(t) + \bar{B}_{\sigma(t)}u(t) + \bar{B}_{d\sigma(t)}d(t) + B_{f\sigma(t)}f(t) \\ y(t) &= \bar{C}_{\sigma(t)}x(t) + \bar{D}_{\sigma(t)}u(t) + \bar{D}_{d\sigma(t)}d(t) + D_{f\sigma(t)}f(t) \end{aligned} \tag{1}$$

where $x(t) \in \Re^n$ is the state vector, $u(t) \in \Re^r$ is the control input vector, $y(t) \in \Re^m$ is the output vector, $d(t) \in \Re^p$ is the unknown inputs (disturbances, noise) vector, $f(t) \in \Re^q$ is the vector, $\sigma(t)$ is a switching signal which is piecewise constant function $\sigma : [0, \infty) \longrightarrow \rho$. Such a function σ has a finite number of switching times and takes a constant value on every interval between two consecutive switching times. The role of $\sigma(t)$ is to specify, at each instant t, the index $\sigma(t) \in \rho$ of the active subsystem. $\bar{A}_{\sigma(t)}, \bar{B}_{\sigma(t)}, \bar{C}_{\sigma(t)}, \bar{D}_{\sigma(t)}, \bar{B}_{d\sigma(t)}, \bar{D}_{d\sigma(t)}, B_{f\sigma(t)}, D_{f\sigma(t)}$ are the systems, disturbances and fault coupling matrices with appropriate dimensions and
$\bar{A}_{\sigma(t)} = A_{\sigma(t)} + \Delta A_{\sigma(t)}, \bar{B}_{\sigma(t)} = B_{\sigma(t)} + \Delta B_{\sigma(t)}$,
$\bar{C}_{\sigma(t)} = C_{\sigma(t)} + \Delta C_{\sigma(t)}, \bar{D}_{\sigma(t)} = D_{\sigma(t)} + \Delta D_{\sigma(t)}$,
$\bar{B}_{d\sigma(t)} = B_{d\sigma(t)} + \Delta B_{d\sigma(t)}, \bar{D}_{d\sigma(t)} = D_{d\sigma(t)} + \Delta D_{d\sigma(t)}$
where, $\Delta A_{\sigma(t)}, \Delta B_{\sigma(t)}, \Delta C_{\sigma(t)}, \Delta D_{\sigma(t)}, \Delta B_{d\sigma(t)}, \Delta D_{d\sigma(t)}$ are norm bounded uncertainties with following definition

$$\begin{aligned} \Delta A_{\sigma(t)} &= E_{\sigma(t)}\Delta(t)G_{\sigma(t)}, \Delta B_{\sigma(t)} = E_{\sigma(t)}\Delta(t)H_{\sigma(t)} \\ \Delta B_{d\sigma(t)} &= E_{\sigma(t)}\Delta(t)J_{\sigma(t)}\Delta C_{\sigma(t)} = F_{\sigma(t)}\Delta(t)G_{\sigma(t)} \\ \Delta D_{\sigma(t)} &= F_{\sigma(t)}\Delta(t)H_{\sigma(t)}\Delta D_{d\sigma(t)} = F_{\sigma(t)}\Delta(t)J_{\sigma(t)} \end{aligned}$$

where $E_{\sigma(t)}, F_{\sigma(t)}, G_{\sigma(t)}, H_{\sigma(t)}, J_{\sigma(t)}$ are known matrices of appropriate dimensions and $\Delta(t)$ is unknown but norm bounded $\Delta(t)^T\Delta(t) \leq I$. We denote the association of these matrices with particular switching signal instant $\sigma(t) = i$ by $A_{\sigma(t)} = A_i$, where $i = 1, 2, ...N$, number of subsystems involved. In order to generate residual signal, the following switched fault detection filter model is used as residual generator.

$$\begin{aligned} \dot{\hat{x}}(t) &= A_{\sigma'(t)}\hat{x}(t) + B_{\sigma'(t)}u(t) - L_{\sigma'(t)}(y(t) - \hat{y}(t)) \\ \hat{y}(t) &= C_{\sigma'(t)}\hat{x}(t) + D_{\sigma'(t)}u(t) \\ r(t) &= H_{\sigma'(t)}(y(t) - \hat{y}(t)) \end{aligned} \tag{2}$$

Where $L_{\sigma'(t)} \in \Re^{n\times n}$ and $H_{\sigma'(t)} \in \Re^{q\times n}$ are the parameters of the filter to be designed with respect to the each subsystem $i \in \{1, 2, ...N\}$. Similar to the system, the switching between different modes of the filter depends on the switching signal $\sigma'(t)$, shown in Fig. 1. The phenomenon of asynchronous switching is shown in the Fig. 2. It is easy to see that each filter lags by some time Δ_i to its corresponding subsystem.

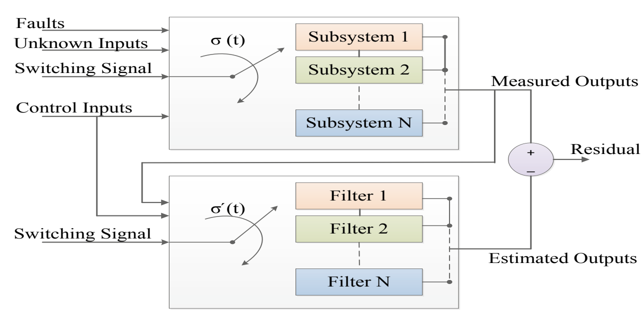

Fig. 1. Switched system and filters

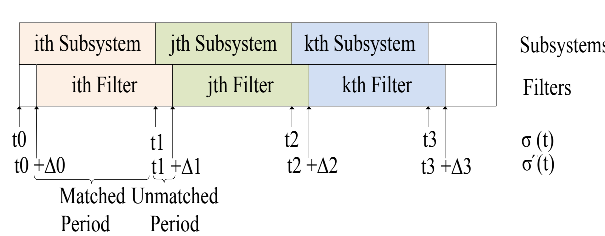

Fig. 2. Asynchronous switching-filters and subsystems

2.3 Definitions and Lemmas

Lemma 1. Liberzon (2003); A switched system

$$\begin{aligned} \dot{x}(t) &= A_i x(t) + B_i u(t) \\ y(t) &= C_i x(t) + D_i u(t) \end{aligned}$$

where $i \in \{1, 2, ...N\}$ is said to be globally asymptotically stable with average dwell time (ADT)

$$\tau_\alpha > \tau_\alpha{}^* = \frac{ln\mu}{\alpha}$$

and satisfies the H_∞ performance with index no greater than $\gamma = max(\gamma_i)$, if there exists Lyapunov functions $V_i(x(t))$ $\forall i \in \{1, 2, ...N\}$ such that

- $V_i(x(t)) \leq \mu V_j(x(t))$
- $\dot{V}_i(x(t)) \leq -\alpha V_i(x(t)) - y^T(t)y(t) + \gamma_i{}^2 u^T(t)u(t)$ $\forall i, j \in \{1, 2, ...N\}$ and $i \neq j$

Lemma 2. Ding (2008); Let G, L, E and $F(t)$ are real matrices of appropriate dimensions with $F(t)$ being a matrix function and $F(t)^T F(t) \leq I$ then for any $\epsilon > 0$

$$LF(t)E + E^T F^T(t) L^T \leq \frac{1}{\epsilon} LL^T + \epsilon E^T E$$

2.4 H_-/H_∞ FDF and Fault Isolation

Each subsystem of the considered switched system is Linear Time Invariant (LTI) and therefore each subsystem individually can be represented by a transfer function $G_i(s) = (A_i, B_i, C_i, D_i)$ $\forall i \in \{1, 2, ..., N\}$. It is intended to design FDF such that there is,

(1) $\| r_f(t) \|_2 \geq \eta_i \| f(t) \|_2$ $\forall i \in \{1, 2, ...N\}$ and
(2) $\| r_d(t) \|_2 \leq \gamma_i \| d(t) \|_2$ $\forall i \in \{1, 2, ...N\}$.

Residual signal $R(s)$ during each mode is given as

$$R(s) = F_i(s)(G_{di}(s)d(s) + G_{fi}(s)f(s))$$

where, $F_i(s) = (A_i + L_iC_i, L_i, H_iC_i, H_i)$ is the post filter for each subsystem of the switched system, for further details, see Li et al. (2006).

3. SOLUTION TO THE H_-/H_∞PROBLEM

H_-/H_∞ solution is a sort of compromise between maximizing sensitivity level and minimizing disturbance attenuation level. To this end, different variants may exist. Details of the our derived results are given in following Theorem 1.

Theorem 1. Given System (1),and suppose that $\Delta^T(t)\Delta(t) \leq 1, \alpha > 0, \rho > 0$, $\mu_1 \geq 0$, $\mu_2 \geq 0$, and $\eta_i \geq 1$, if there exist $P_i > 0, P_{ij} > 0$ for $i \neq j$, and $i, j \in N$ such that the following inequalities

$$\begin{bmatrix} P_{11ij} & P_{12ij} \\ * & P_{22ij} \end{bmatrix} > 0 \quad , \begin{bmatrix} P_{11i} & P_{12i} \\ * & P_{22i} \end{bmatrix} > 0 \tag{3}$$

$$\begin{bmatrix} P_{11j} & P_{12j} \\ * & P_{22j} \end{bmatrix} \leq \mu_1 \begin{bmatrix} P_{11ij} & P_{12ij} \\ * & P_{22ij} \end{bmatrix} \tag{4}$$

$$\begin{bmatrix} P_{11ij} & P_{12ij} \\ * & P_{22ij} \end{bmatrix} \leq \mu_2 \begin{bmatrix} P_{11i} & P_{12i} \\ * & P_{22i} \end{bmatrix} \tag{5}$$

$$\begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} & \Psi_{14} & C_i{}^T D_{fi}{}^{-T} \eta_i{}^T & \Psi_{16} \\ * & \Psi_{22} & \Psi_{23} & \Psi_{24} & -C_i{}^T D_{fi}{}^{-T} \eta_i{}^T & \Psi_{26} \\ * & * & \Psi_{33} & \epsilon H_i^T J_i & 0 & 0 \\ * & * & * & \Psi_{44} & D_{di}{}^T D_{fi}{}^{-T} \eta_i{}^T & 0 \\ * & * & * & * & -I & \Psi_{56} \\ * & * & * & * & * & -\epsilon I \end{bmatrix} < 0 \tag{6}$$

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & C_j{}^T D_{fi}{}^{-T} \eta_i{}^T & \Omega_{16} \\ * & \Omega_{22} & \Omega_{23} & \Omega_{24} & -C_i{}^T D_{fi}{}^{-T} \eta_i{}^T & \Omega_{26} \\ * & * & \Omega_{33} & \epsilon H_j^T J_j & \Omega_{35} & 0 \\ * & * & * & \Omega_{44} & D_{dj}{}^T D_{fi}{}^{-T} \eta_i{}^T & 0 \\ * & * & * & * & -I & \Omega_{56} \\ * & * & * & * & * & -\epsilon I \end{bmatrix} < 0 \tag{7}$$

hold, where

$$\Psi_{11} = A_i^T P_{11i} + C_i^T D_{fi}{}^{-T} B_{fi}{}^T P_{12i}{}^T + P_{11i} A_i + P_{12i} B_{fi} D_{fi}{}^{-1} C_i + \epsilon G_i^T G_i + \alpha P_{11i}$$
$$\Psi_{12} = A_i^T P_{12i} + C_i^T D_{fi}{}^{-T} B_{fi}{}^T P_{22i}{}^T + P_{12i} A_i - P_{12i} B_{fi} D_{fi}{}^{-1} C_i + \alpha P_{12i}$$
$$\Psi_{13} = P_{11i} B_i + P_{12i} B_i + \epsilon G_i^T H_i$$
$$\Psi_{14} = P_{11i} B_{di} + P_{12i} B_{fi} D_{fi}{}^{-1} D_{di} + \epsilon G_i^T J_i$$
$$\Psi_{16} = P_{11i} E_i + P_{12i} B_{fi} D_{fi}{}^{-1} F_i$$
$$\Psi_{22} = A_i^T P_{22i} - C_i^T D_{fi}{}^{-T} B_{fi}{}^T P_{22i} + P_{22i} A_i - P_{22i} B_{fi} D_{fi}{}^{-1} C_i + \alpha P_{22i}$$
$$\Psi_{23} = P_{12i}{}^T B_i + P_{22i} B_i, \Psi_{33} = \epsilon H_i^T H_i - \gamma_i^2$$
$$\Psi_{24} = P_{12i}{}^T B_{di} + P_{22i} B_{fi} D_{fi}^{-1} D_{di}$$
$$\Psi_{26} = P_{12i}{}^T E_i + P_{22i} B_{fi} D_{fi}^{-1} F_i$$
$$\Psi_{44} = \epsilon J_i^T J_i - \gamma_i^2, \Psi_{56} = -\eta_i D_{fi}^{-1} F_i$$
$$\Omega_{11} = A_j^T P_{11ij} + C_j^T D_{fi}{}^{-T} B_{fi}{}^T P_{12ij}{}^T + P_{11ij} A_j + P_{12ij} B_{fi} D_{fi}{}^{-1} C_j + \epsilon G_j^T G_j - \rho P_{11ij}$$
$$\Omega_{12} = A_j^T P_{12ij} + C_j^T D_{fi}{}^{-T} B_{fi}{}^T P_{22ij}{}^T + P_{12ij} A_i - P_{12ij} B_{fi} D_{fi}{}^{-1} C_i - \rho P_{12ij}$$
$$\Omega_{13} = P_{11ij} B_j + P_{12ij} B_i + P_{12ij} B_{fi} D_{fi}{}^{-1} D_j - P_{12ij} B_{fi} D_{fi}{}^{-1} D_i + \epsilon G_j^T H_j$$
$$\Omega_{14} = P_{11ij} B_{dj} + P_{12ij} B_{fi} D_{fi}{}^{-1} D_{dj} + \epsilon G^T J$$
$$\Omega_{16} = P_{11ij} E_j + P_{12ij} B_{fi} D_{fi}{}^{-1} F_j$$
$$\Omega_{22} = A_i^T P_{22ij} - C_i^T D_{fi}{}^{-T} B_{fi}{}^T P_{22ij} + P_{22ij} A_i - P_{22ij} B_{fi} D_{fi}{}^{-1} C_i - \rho P_{22ij}$$
$$\Omega_{23} = P_{12ij}{}^T B_j + P_{22ij} B_i + P_{22ij} B_{fi} D_{fi}{}^{-1} Dj - P_{22ij} B_{fi} D_{fi}{}^{-1} Di, \Omega_{33} = \epsilon H_j^T H_j - \gamma_{ij}^2$$
$$\Omega_{24} = P_{12ij}{}^T B_{dj} + P_{22ij} B_{fi} D_{fi}^{-1} D_{dj}$$
$$\Omega_{26} = P_{12ij}{}^T E_j + P_{22ij} B_{fi} D_{fi}^{-1} F_j$$
$$\Omega_{35} = D_j^T D_{fi}{}^{-T} \eta_i{}^T - D_i^T D_{fi}{}^{-T} \eta_i{}^T$$
$$\Omega_{44} = \epsilon J_j^T J_j - \gamma_{ij}{}^2 I, \Omega_{56} = -\eta_i D_{fi}^{-1} F_j$$

then, switched system (1) and detection filter (2) are globally asymptotically stable in augmented form, the H_-/H_∞ filter design objective is met, and occurring faults are isolated for any switching signal with (ADT) $\tau_a \geq \tau_a{}^* = \frac{\ln \mu}{\alpha}$. Further, objective post-filer can be obtained by $F_i(s) = (A_i + L_iC_i, L_i, H_iC_i, H_i) \in \Re H_\infty^{q \times m}$ And parameters of detection filter are given by

$$L_i = -B_{fi} D_{fi}{}^{-1}, H_i = \eta_i D_{fi}{}^{-1}$$

Proof 1. we require that $\|F_i G_{fi}\|_- \geq \eta_i$ and also we know that

$$F_i(s) G_{fi}(s) = (A_i + L_iC_i, B_{fi} + L_i D_{fi}, H_i C_i, H_i D_{fi}) \in \Re H_\infty^{q \times m}$$

To achieve the above mentioned objective, we should have $F_iG_{fi} = \eta_i I$, so that $\|F_iG_{fi}\|_- \geq \eta_i \ \forall i \in \{1,2,...N\}$ This can be achieved easily by setting

$$B_{fi} + L_iD_{fi} = 0 \text{ and } H_iD_{fi} = \eta_i I_q$$

From these two equations, we can find $L_i = -B_{fi}{D_{fi}}^{-1}$, $H_i = \eta_i {D_{fi}}^{-1}$ Now, for the desired L_i and H_i the only remaining part of the problem is to find out H_∞ norm of $F_iG_{di}, \forall i \in \{1,2,...N\}$. To this end, we use Lemma 1 under asynchronous paradigm during matched and unmatched period.

Stability Analysis during Matched Period:
During the matched period, ith subsystem and ith filter are in operation, see Fig.2. We augment the switched system (1) and detection filter (2) into the following compact representation during matched period

$$\begin{aligned}\dot{\tilde{x}}(t) &= (\tilde{A}_i + \Delta\tilde{A}_i)\tilde{x}(t) + \breve{B}_{fi}f(t) + (\tilde{B}_i + \Delta\tilde{B}_i))\omega(t)\\ r(t) &= (\tilde{C}_i + \Delta\tilde{C}_i)\tilde{x}(t) + \breve{D}_{fi}f(t) + (\tilde{D}_i + \Delta\tilde{D}_i)\omega(t)\end{aligned} \quad (8)$$

where, $\tilde{x}(t) = \begin{bmatrix}x(t)^T & \hat{x}(t)^T\end{bmatrix}^T$, $\omega(t) = \begin{bmatrix}u(t)^T & d(t)^T\end{bmatrix}^T$

$$\tilde{A}_i = \begin{bmatrix}A_i & 0\\ -L_iC_i & A_i + L_iC_i\end{bmatrix}, \Delta\tilde{A}_i = \begin{bmatrix}\Delta A_i & 0\\ -L_i\Delta C_i & 0\end{bmatrix}$$
$$\tilde{B}_i = \begin{bmatrix}B_i & B_{di}\\ B_i & -L_iD_{di}\end{bmatrix} \Delta\tilde{B}_i = \begin{bmatrix}\Delta B_i & \Delta B_{di}\\ -L_i\Delta D_i & -L_i\Delta D_{di}\end{bmatrix}$$
$$\tilde{C}_i = [H_iC_i \ -H_iC_i], \Delta\tilde{C}_i = [H_i\Delta C_i \ 0]$$
$$\tilde{D}_i = [0 \ H_iD_{di}], \Delta\tilde{D}_i = [H_i\Delta D_i \ H_i\Delta D_{di}]$$
$$\breve{B}_{fi} = \begin{bmatrix}B_{fi}\\ 0\end{bmatrix}, \breve{D}_{fi} = [H_iD_{fi}]$$

Using Lemma1, during matched period,

$$V_i(\tilde{x}(t)) \leq \mu V_j(\tilde{x}(t)) \quad (9)$$

$$\dot{V}_i(\tilde{x}(t)) \leq -\alpha V_i(\tilde{x}(t)) - r^Tr(t) + \gamma_i^2\omega(t)^T\omega(t) \quad (10)$$

Considering the following Lyapunov function,

$$V_i(\tilde{x}(t)) = \tilde{x}^T(t)P_i\tilde{x}(t) \quad (11)$$

Differentiating (11)

$$\dot{V}_i(\tilde{x}(t)) = \dot{\tilde{x}}^T(t)P_i\tilde{x}(t) + \tilde{x}^T(t)P_i\dot{\tilde{x}}(t) \quad (12)$$

Substituting $r(t)$ and (11), (12), in (10), and also

$$\breve{A} = (\tilde{A}_i + \Delta\tilde{A}_i), \breve{B} = (\tilde{B}_i + \Delta\tilde{B}_i)$$
$$\breve{C} = (\tilde{C}_i + \Delta\tilde{C}_i), \breve{D} = (\tilde{D}_i + \Delta\tilde{D}_i)$$

following inequality is obtained

$$\begin{aligned}&\dot{\tilde{x}}^T(t)P_i\tilde{x}(t) + \tilde{x}^T(t)P_i\dot{\tilde{x}}(t) + \left[\breve{C}_i\tilde{x}(t) + \breve{D}_i\omega(t)\right]^T\\ &\left[\breve{C}_i\tilde{x}(t) + \breve{D}_i\omega(t)\right] \leq -\alpha\tilde{x}^T(t)P_i\tilde{x}(t) + \gamma_i^2\omega(t)^T\omega(t)\end{aligned} \quad (13)$$

Further, substituting the expression for $\dot{\tilde{x}}(t)$

$$\begin{aligned}&\left[\breve{A}_i\tilde{x}(t) + \breve{B}_i\omega(t)\right]^T P_i\tilde{x}(t) + \tilde{x}^T(t)P_i\left[\breve{A}_i\breve{x}(t) + \breve{B}_i\omega(t)\right]\\ &+(\tilde{x}^T(t)\breve{C}_i^T + \omega^T(t)\breve{D}_i^T)(\breve{C}_i\tilde{x}(t) + \breve{D}_i\omega(t))\\ &\leq -\alpha\tilde{x}^T(t)P_i\tilde{x}(t) + \gamma_i^2\omega(t)^T\omega(t))\end{aligned} \quad (14)$$

(14) can be written easily in following form

$$\begin{aligned}&\left[\tilde{x}^T(t)\breve{A}_i^TP_i + \omega^T(t)\breve{B}_i^TP_i\right]\tilde{x}(t) + \tilde{x}^T(t)P_i\breve{A}_i\tilde{x}(t)\\ &+\tilde{x}^T(t)P_i\breve{B}_i\omega(t) + \tilde{x}^T(t)\breve{C}_i^T\breve{C}_i\tilde{x}(t)\\ &+\tilde{x}^T(t)\breve{C}_i^T\breve{D}_i\omega(t) + \omega^T(t)\breve{D}_i^T\breve{C}_i\tilde{x}(t)\\ &+\omega^T(t)\breve{D}_i^T\breve{D}_i\omega(t)) + \alpha\tilde{x}^T(t)P_i\tilde{x}(t)\\ &-{\gamma_i}^2\omega^T(t)\omega(t) \leq 0\end{aligned} \quad (15)$$

Further, the above inequality can be written as

$$\begin{bmatrix}\tilde{x}^T(t) & \omega^T(t)\end{bmatrix} M \begin{bmatrix}\tilde{x}(t)\\ \omega(t)\end{bmatrix} \leq 0 \quad (16)$$

Where,

$$M = \begin{bmatrix}{\breve{A}_i}^TP_i + P_i\breve{A}_i + {\breve{C}_i}^T\breve{C}_i + \alpha P_i & P_i\breve{B}_i + {\breve{C}_i}^T\breve{D}_i\\ * & {\breve{D}_i}^T\breve{D}_i - \gamma_i^2 I\end{bmatrix}$$

for (16) to hold, it is required that

$$M < 0 \quad (17)$$

After Schur's compliment is applied to (17), we get

$$\begin{bmatrix}{\breve{A}_i}^TP_i + P_i\breve{A}_i + \alpha P_i & P_i\breve{B}_i & {\breve{C}_i}^T\\ * & -\gamma_i^2 I & {\breve{D}_i}^T\\ * & * & -I\end{bmatrix} < 0 \quad (18)$$

To separate the uncertainties terms, splitting the LMI (18)

$$\begin{aligned}&\begin{bmatrix}{\tilde{A}_i}^TP_i + P_i\tilde{A}_i + \alpha P_i & P_i\tilde{B}_i & {\tilde{C}_i}^T\\ * & -\gamma_i^2 I & {\tilde{D}_i}^T\\ * & * & -I\end{bmatrix} +\\ &\begin{bmatrix}\Delta{\tilde{A}_i}^TP_i + P_i\Delta\tilde{A}_i & P_i\Delta\tilde{B}_i & \Delta{\tilde{C}_i}^T\\ * & 0 & \Delta{\tilde{D}_i}^T\\ * & * & 0\end{bmatrix} < 0\end{aligned} \quad (19)$$

second matrix in above inequality can be written into following following form

$$\begin{aligned}&\begin{bmatrix}\bar{E}1 & \bar{E}2 & \bar{E}3 & 0 & 0 & 0 & -H_iF\end{bmatrix}^T \Delta(t)\,[0 \ G \ 0 \ H \ J \ 0 \ 0]\\ &+[\begin{bmatrix}\bar{E}1 & \bar{E}2 & \bar{E}3 & 0 & 0 & 0 & -H_iF\end{bmatrix}^T \Delta(t)\,[0 \ G \ 0 \ H \ J \ 0 \ 0]]^T\end{aligned} \quad (20)$$

According to Lemma 2, we know that (18) holds if there exists $\epsilon > 0$ so that

$$\begin{aligned}&\begin{bmatrix}{\tilde{A}_i}^TP_i + P_i\tilde{A}_i + \alpha P_i & P_i\tilde{B}_i & {\tilde{C}_i}^T\\ * & -\gamma_i^2 I & {\tilde{D}_i}^T\\ * & * & -I\end{bmatrix}\\ &+\frac{1}{\epsilon}\begin{bmatrix}\bar{E}1 & \bar{E}2 & \bar{E}3 & 0 & 0 & 0 & -H_iF\end{bmatrix}^T\begin{bmatrix}\bar{E}1 & \bar{E}2 & \bar{E}3 & 0 & 0 & 0 & -H_iF\end{bmatrix}\\ &+\epsilon[0 \ G \ 0 \ H \ J \ 0 \ 0]^T\,[0 \ G \ 0 \ H \ J \ 0 \ 0] < 0\end{aligned} \quad (21)$$

Finally, by applying Schur's complement again,

$$\begin{bmatrix}\psi & P_i\tilde{B} + \epsilon\bar{G}^T\bar{H} & \tilde{C}_i^T & P_i\bar{E}\\ * & -\gamma_i^2 I + \epsilon\bar{H}^T\bar{H} & \tilde{D}_i^T & 0\\ * & * & -I & -H_iF\\ * & * & * & -\epsilon I\end{bmatrix} < 0 \quad (22)$$

where ,$\psi = {\tilde{A}_i}^TP_i + P_i\tilde{A}_i + \alpha P_i + \epsilon\bar{G}^T\bar{G}, \bar{G} = [G0], \bar{H} = [HJ], \bar{E}^T = [E^T(-L_iF)^T]$
Then, substituting, L_i and H_i in (22), LMI (6) of the Theorem 1 is obtained.

Stability Analysis during Unmatched Period: During the unmatched period, jth subsystem and ith filter are switched together, see Fig.2. We augment the switched system (1) and detection filter (2) into the following compact representation during unmatched period

$$\dot{\tilde{x}}(t) = (\tilde{A_{ij}} + \Delta\tilde{A_{ij}})\tilde{x}(t) + \breve{B}_{fij} f(t) + (\tilde{B_{ij}} + \Delta\tilde{B_{ij}}))\omega(t)$$
$$r(t) = (\tilde{C_{ij}} + \Delta\tilde{C_{ij}})\tilde{x}(t) + \breve{D}_{fij} f(t) + (\tilde{D_{ij}} + \Delta\tilde{D_{ij}})\omega(t) \tag{23}$$

where, $\tilde{x}(t) = \left[x(t)^T \;\; \hat{x}(t)^T\right]^T$, $\omega(t) = \left[u(t)^T \;\; d(t)^T\right]^T$

$$\tilde{A_{ij}} = \begin{bmatrix} A_j & 0 \\ -L_iC_j & A_i + L_iC_i \end{bmatrix}, \Delta\tilde{A_{ij}} = \begin{bmatrix} \Delta A_j & 0 \\ -L_i\Delta C_j & 0 \end{bmatrix}$$
$$\tilde{B_{ij}} = \begin{bmatrix} B_j & B_{dj} \\ B_i - L_iD_j + L_iD_i & -L_iD_{dj} \end{bmatrix}$$
$$\Delta\tilde{B_i} = \begin{bmatrix} \Delta B_j & \Delta B_{dj} \\ -L_i\Delta D_j & -L_i\Delta D_{dj} \end{bmatrix}$$
$$\tilde{C_{ij}} = [H_iC_j \;\; -H_iC_i], \Delta\tilde{D_{ij}} = [H_i\Delta D_j \;\; H_i\Delta D_{dj}]$$
$$\tilde{D_{ij}} = [H_iD_j - H_iD_i \;\; H_iD_{dj}], \Delta\tilde{C_{ij}} = [H_i\Delta C_j \;\; 0]$$
$$\breve{B}_{fij} = \begin{bmatrix} B_{fj} \\ 0 \end{bmatrix}, \breve{D}_{fij} = [H_iD_{fj}]$$

During unmatched period, using Lemma 1

$$V_{ij}(\tilde{x}(t)) \le \mu v_j(\tilde{x}(t)) \tag{24}$$
$$\dot{V_{ij}}(\tilde{x}(t)) \le \rho V_{ij}(\tilde{x}(t)) - r^T r(t) + \gamma_i^2 \omega(t)^T\omega(t) \tag{25}$$

where $i \neq j$ and $i, j \in N$ During unmatched period, the following Lyapunov function is used

$$V_{ij}(\tilde{x}(t)) = \tilde{x}^T(t)P_{ij}\tilde{x}(t) \tag{26}$$

To derive further the results of Theorem, we apply the same procedure as it is done earlier for matched period. To this end, we skip the next steps which result in inequality (7) of Theorem 1. ■

4. ADAPTIVE THRESHOLD COMPUTATION AND RESIDUAL EVALUATION

After successful residual generation, the next step is to evaluate further the residual signal. In this work following residual evaluation function is used

$$J_{RMS} = \| r(t) \|_{RMS} = (\tfrac{1}{T}\int_t^{t+T} \| r(\tau) \|^2)^{\frac{1}{2}} d\tau$$

where, T is the evaluation window.

Along with residual evaluation function, the threshold computation is also required for efficient detection of faults. Threshold value is the maximum influence of unknown inputs (disturbances, noises) and model uncertainties on the residual signal in the absence of faults. Threshold can also be of different types like fixed, adaptive, or dynamic Abdo et al. (2011); Khan and Ding (2011) In this research the following adaptive threshold is employed,

$$J_{adap.th,\sigma(t)RMS,2} = \frac{\gamma_\sigma(t)}{\sqrt{T}}(\delta_{d,2,\sigma(t)} + \|u\|_{2,\sigma(t)})$$

where, $\gamma_\sigma(t)$ is mode dependent robustness factor, $\delta_{d,2,\sigma(t)}$ is norm bounded, mode dependent disturbance acting on the corresponding mode which can be found set off-line and $\|u\|_{2,\sigma(t)}$ is input which is time dependent parameter, computed on-line. Finally, decision about the presence of fault in the system is made by the following logic

- $J_{RMS} < J_{th,RMS,2} \Longrightarrow$ No FAULT
- $J_{RMS} > J_{th,RMS,2} \Longrightarrow$ Detected FAULT

5. CASE STUDY: APPLICATION TO BOOST-CONVERTER UNIT

In this section simulation results for the case study of boost-converter unit are presented. The Simulation time is setup for 30s, such that, subsystem 1 is activated when $\sigma(t) = 1$ and subsystem 2 when $\sigma(t) = 0$. Two faults, $f_1(t)$ = -1 short duration fault, and $f_2(t) = 1$ of biased type are simulated. $f_1(t)$ is considered as battery fault while $f_2(t)$ as converter fault in capacitor. Details of temporal switching behavior of subsystems, filters and faults can be seen in Fig. 4.

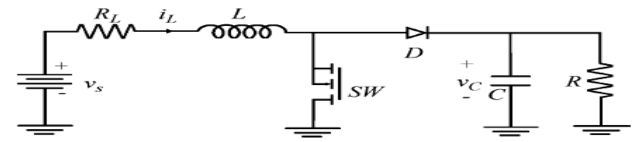

Fig. 3. Boost converter circuit- Tanwani et al. (2011)

$$A_1 = \begin{bmatrix} -\frac{R_l}{L} & 0 \\ 0 & -\frac{1}{RC} \end{bmatrix}, B_1 = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}$$
$$A_2 = \begin{bmatrix} -\frac{R_L}{L} & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix}, B_2 = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}$$

Next, we discuss the results, based on following design parameters of boost converter shown in Fig.3 Tanwani et al. (2011). R_L = 0.2 ohms, L = 0.05 mH, C = 200 μ F, R = 24 ohms, and v_s = 12 volts. Here, we assume that both state variables i_L, inductor current, and v_C, capacitor voltage, are available for measurement, and voltage v_s is known. The battery converter unit is a switched system, operating in two modes. In mode 1, the transistor switch SW is CLOSED and the diode switch D is OPENED, which corresponds to system dynamics A_1, B_1. In mode 2, the transistor switch SW is OPENED and the diode switch D is CLOSED, which corresponds to system dynamics A_2, B_2,

To this end, the dynamics of boost converter in matrices form are given below

$$A_1 = \begin{bmatrix} -4000 & 0 \\ 0 & -208.33 \end{bmatrix}, B_1 = \begin{bmatrix} 20000 \\ 0 \end{bmatrix}, C_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$B_{d1} = \begin{bmatrix} -0.1 & 0.03 \\ -0.2 & 0.1 \end{bmatrix}, D_{d1} = \begin{bmatrix} 0.02 & -0.1 \\ -0.01 & 0.02 \end{bmatrix}, D_1 = \begin{bmatrix} 0.3 \\ 0 \end{bmatrix},$$
$$B_{f1} = \begin{bmatrix} 20000 & 0 \\ 0 & 0 \end{bmatrix}, D_{f1} = \begin{bmatrix} 0.3 & 0 \\ 0 & 1 \end{bmatrix}, E_1 = \begin{bmatrix} 0.2 \\ 0.35 \end{bmatrix},$$
$$F_1 = \begin{bmatrix} 0.1 \\ 0.25 \end{bmatrix}, G_1 = [0.2 \;\; 0.3], H_1 = [0.2], J_1 = [0.3 \;\; 0.2]$$
$$A_2 = \begin{bmatrix} -4000 & -20000 \\ 5000 & -208.33 \end{bmatrix}, B_2 = \begin{bmatrix} 20000 \\ 0 \end{bmatrix}, C_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$B_{d2} = \begin{bmatrix} -0.01 & -0.03 \\ 0.1 & -0.16 \end{bmatrix}, D_{d2} = \begin{bmatrix} 0.11 & 0.3 \\ 0.2 & -0.01 \end{bmatrix}, D_2 = \begin{bmatrix} 0.2 \\ 0 \end{bmatrix},$$
$$B_{f2} = \begin{bmatrix} 20000 & 0 \\ 0 & 0 \end{bmatrix}, D_{f2} = \begin{bmatrix} 0.2 & 0 \\ 0 & 1 \end{bmatrix}, E_2 = \begin{bmatrix} 0.3 \\ 0.25 \end{bmatrix}$$

$$F_2 = \begin{bmatrix} 0.2 \\ 0.25 \end{bmatrix}, G_2 = [0.3\ \ 0.2], H_2 = [0.3], J_2 = [0.2\ \ 0.35]$$

By solving LMIs (3)-(7) of Theorem 1, we find the disturbance attenuation levels are $\gamma_1 = 0.3873, \gamma_2 = 0.7071$ and the filter parameters are found to be

$$L1 = \begin{bmatrix} -66667 & 0 \\ 0 & 0 \end{bmatrix}, H1 = \begin{bmatrix} 3.3333 & 0 \\ 0 & 1.0000 \end{bmatrix}$$

$$L2 = \begin{bmatrix} -100000 & 0 \\ 0 & 0 \end{bmatrix}, H2 = \begin{bmatrix} 5.0000 & 0 \\ 0 & 1.0000 \end{bmatrix}$$

Then, the battery-converter system is simulated as in Fig. 4. The residual signals for the system in absence and presence of faults are depicted in Fig. 5. Here it is easy to see that $f_1(t)$ affects only the $r_1(t)$ whereas $f_2(t)$ has influence exclusively to $r_2(t)$. In this way, not only both faults are successfully detected but also isolated (located). Finally, to avoid false alarms, residuals are evaluated by RMS and adaptive thresholds are set according to section IV discussion, which are depicted in Fig.6. For residual evaluation window size T is set to 10. Fault is detected properly when evaluated residuals cross their respective adaptive thresholds values just after the occurrence of faults at time 3s and 8s.

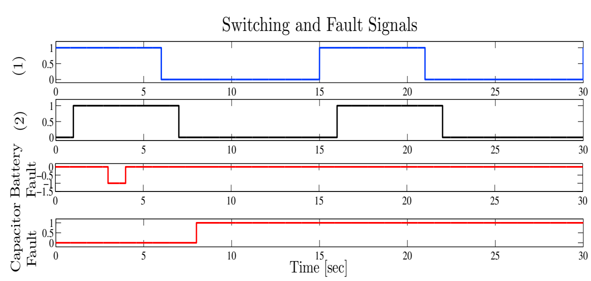

Fig. 4. (1) and (2): Switching signals

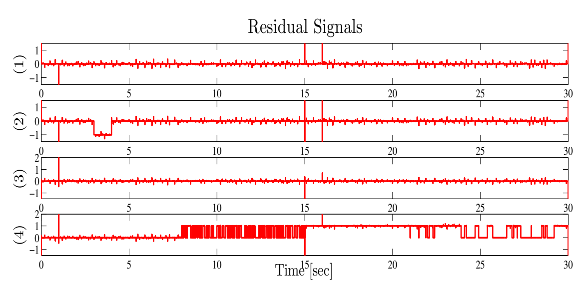

Fig. 5. (1): $r_1(t)$ without fault (2): $r_1(t)$ with fault $f_1(t)$ (3): $r_2(t)$ without fault (4): $r_2(t)$ with fault $f_2(t)$

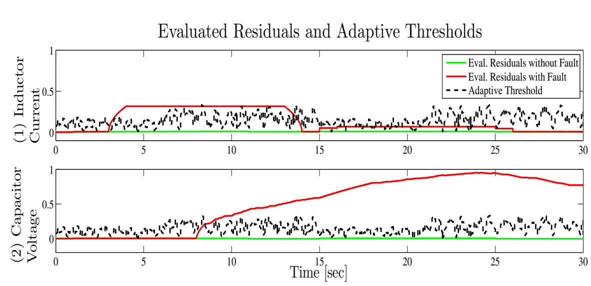

Fig. 6. Evaluated residuals and thresholds

6. CONCLUSION

In this paper, the problem of fault detection and isolation for continuous-time uncertain switched system has been addressed. Due to nature of power converter system as switched system, fault detection filter is assumed to be switching asynchronously with modes of converter. Solution is provided in the form of a mixed H_-/H_∞ fault detection filter. To improve the fault detection adaptive threshold is employed. Results show the effectiveness of proposed strategy.

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