Design of Fault Detection and Isolation Filter for Switched Control Systems Under Asynchronous Switching

Muhammad Taskeen Raza, Abdul Qayyum Khan, Ghulam Mustafa, and Muhammad Abid

***Abstract*— The problem of fault detection (FD) and isolation, in the presence of disturbances and noise, for continuous-time linear switched control systems is addressed in this paper. The residual generator is proposed, which is based on asynchronously switching filters, where asynchronous means that there is a lag between switching of filters and subsystems. To address the issue, the FD filter problem is formulated as a mixed H_-/H_∞ filtering problem. In the proposed H_-/H_∞ technique, residual is generated such that it is sensitive to faults and robust against process disturbances and measurement noise. In addition, the proposed filter has prominence of granting fault isolation capability along with FD. To deal with the major issue of asynchronous switching, during matched and unmatched time of switched systems, a piecewise Lyapunov function along with average dwell time scheme is employed, and sufficient conditions are derived in terms of linear matrix inequalities. To simplify the application procedure, an algorithm is also presented in the light of the proposed framework. Then, a solution is designed for two different case studies of highly maneuverable aircraft technology and boost–buck power converter. Finally, the designed filter parameters are simulated to illustrate the efficacy of the proposed framework.**

***Index Terms*— Asynchronous switching, average dwell time (ADT), boost–buck converter, fault detection (FD), fault isolation, FD filter, piecewise Lyapunov function, switched systems.**

I. Introduction

FAULT detection and isolation (FDI) for dynamical systems and processes have been of considerable interest for the past three decades. For this purpose, a great deal of attention has been paid to FDI schemes [1]–[11]. Among various fault detection (FD) approaches, the so-called model-based FD and data-driven FD are receiving considerable attention in recent research. In data-driven framework [5], [8], [12], the FD system design relies on input–output data only to generate symptom signal, which carries the fault information. In the case of the availability of system model, the model-based FD [2], [4], [9] is an attractive choice for efficient detection of faults. The basic idea of model-based FD is to generate a residual signal (symptom signal) by comparing the measured and estimated output signals of the system. One of the problems in this research domain is the coupling of desired faults with unknown inputs (disturbances, noise, and uncertainties), which are the major sources of false alarms in industrial systems. To cope with this problem, the FDI system has to be maximally sensitive to faults occurring in the system and at the same time maximally robust against unknown inputs. To this end, a range of optimization indices have been proposed. Few of those are H_2/H_2 [2], H_∞/H_∞ [4], and H_-/H_∞ [11], where the H_-/H_∞ index is of particular interest in our research, because of its twofold nature for the solution to the said problem. The H_- index takes into account the minimum influence of faults, whereas the H_∞ norm considers the worst case effect of unknown inputs on the residual signal.

On the other hand, due to their significance in theory and practical applications, switched systems have fascinated many researchers. These systems have numerous applications in control of robotics, mechanical systems, automotive industry, aircraft and air traffic control, switched power converters, and in many other fields [13]–[22]. Switched systems are a class of hybrid systems consisting of subsystems, which have either continuous-time or discrete-time dynamics, and a switching signal that governs the activation of any particular subsystem along the trajectory of the switched system at any instant of time. Physically, switching signals are mainly of the following types: 1) time-dependent; 2) state-dependent; 3) autonomous; and 4) controlled switching signals. Furthermore, a combination of the aforementioned types of signals may also occur in a switched system [13]. It is also worth noting that stability problems of switched systems are unique and also complex. Individually stable subsystems may become unstable at system level when switching takes place among them. On the other hand, unstable subsystems may be stabilized by choosing a suitable switching scheme. This critical situation turns into rather severe nature, when a fault occurs in any component of a switched system.

Currently, it is an active research area to explore the problem of fault diagnosis and fault tolerant control (FTC) for switched systems. An FTC system is a control system with the ability to tolerate faults automatically and continue its operation in the event of a fault in some of its components [23]–[25].

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The authors are with the Department of Electrical Engineering, Pakistan Institute of Engineering and Applied Sciences, Islamabad 45650, Pakistan (e-mail: taskeenraza@pieas.edu.pk; aqkhan@pieas.edu.pk; gm@pieas.edu.pk; mabid@pieas.edu.pk).

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FD problem for discrete-time switched systems is considered in [17], while for the continuous-time case, FD problem is considered in [15]. In [16], FD for uncertain discrete time switched systems is considered. FTC problem for continues-time switched systems is studied in [19]. In [26], H_-/H_∞ performance index has been utilized for optimal FD in switched systems. However, all the aforementioned efforts for the problem have assumed that the FD filter is switching with the subsystems in synchronous manner. In this way, the problem is reduced to multiple linear dynamical systems simply. While, in practice, it takes time to identify the particular subsystem active at any instant of time. Therefore, the phenomenon of asynchronous switching between a filter and a subsystem exists in general. In asynchronous switching, there is a lag between filters and subsystems. Although the asynchronous problem has gained attention of FD research community in recent years [14], [22], still it deserves more attention to be paid. In [14], a maximum dwell time technique is employed to deal with the asynchronous switching control problem. In [17], FD problem for discrete switched systems was studied with the assumption that the governing switching signal is unknown. In particular, Du *et al.* [22] motivated us to explore further the asynchronous switching problem in fault diagnosis. In the aforementioned work, H_∞ filtering technique is used to formulate and design the filter structure. In our opinion, in that design technique, it is hard to solve numerically the linear constraint equation and linear matrix inequalities (LMIs), simultaneously see [22, Remark 2]. Furthermore, only the H_∞ norm is utilized in [22]. Here, it is also important to mention a significant effort [27] made to detect and isolate fault in general physical systems.

In this paper, we present a solution to the FDI problem of continuous-time switched systems while taking into account the case of asynchronous switching between filters and subsystems. The switching phenomenon assumes average dwell time (ADT) constraints. The proposed solution has the following outstanding features.

1) The attention is given to the optimal solution in the sense that the effect of unknown inputs on the residual signal is minimized, whereas that of the fault is maximized simultaneously. To this end, H_-/H_∞ optimization index is utilized for the problem of asynchronously switched systems.
2) The Fault Detection Filter (FDF) is proposed, which is formulated in the form of LMIs that are computationally more tractable than that in [22].
3) A significant contribution of the proposed work is to achieve the fault isolation capability in a straightforward way along with FD while investigating the complex asynchronous problem of switched systems.

The rest of this paper is organized as follows. In Section II, preliminaries and problem formulation are given. The solution to the problem is derived in Section III. Section IV is concerned with threshold computation and residual evaluation steps in FD. In Section V, designed filters are applied to switched control systems to show the effectiveness of the results, followed by the conclusion in Section VI.

II. Preliminaries and Problem Formulation

A. Notations

The notations used in this paper are fairly standard such that the set of real matrices with dimensions $m \times n$ is denoted by $R^{m\times n}$. For a matrix $A \in R^{n\times n}$, the transpose of A is represented by A^T, while the transpose inverse of A is denoted by A^{-T}. Positive definiteness is represented by $A = A^T > 0$, whereas $A = A^T < 0$ shows the negative definiteness of matrix $A \in R^{n\times n}$. A stable real rational matrix function of s is denoted by $RH_\infty^{m\times n}$. The pseudoinverse of a matrix $A \in R^{n\times n}$ is represented by $A^\dagger$ such that $A^\dagger A = I_n$, where I_n is the identity matrix. The orthogonal matrix of A is denoted by $A^\perp$ such that $A^\perp A = 0_n$, where 0_n is the representation of zero matrix.

B. Switched System and Fault Detection Filters' Dynamic Models—Asynchronous Switching Problem

Consider the following class of continuous-time switched systems:

$$\begin{cases} \dot{x}(t) = A_{\sigma(t)}x(t)+B_{\sigma(t)}u(t)+B_{d\sigma(t)}d(t) + B_{f\sigma(t)}f(t) \\ y(t) = C_{\sigma(t)}x(t)+D_{\sigma(t)}u(t) + D_{d\sigma(t)}d(t)+D_{f\sigma(t)}f(t) \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^r$ is the control input vector, $y(t) \in R^m$ is the output vector, $d(t) \in R^p$ is the unknown input (disturbances and noise) vector, $f(t) \in R^q$ is the fault vector, and $\sigma(t)$ is a switching signal that is a piecewise constant function $\sigma : [0,\infty) \longrightarrow \rho$. Such a function σ has a finite number of switching times and takes a constant value on every interval between two consecutive switching times. The role of $\sigma(t)$ is to specify, at each instant t, the index $\sigma(t) \in \rho$ of the active subsystem. Also, $A_{\sigma(t)}$, $B_{\sigma(t)}$, $C_{\sigma(t)}$, $D_{\sigma(t)}$, $B_{d\sigma(t)}$, $D_{d\sigma(t)}$, $B_{f\sigma(t)}$, and $D_{f\sigma(t)}$ are the systems, disturbances, and fault coupling matrices with appropriate dimensions. We denote the association of these matrices with a particular switching signal instant $\sigma(t) = i$ by $A_{\sigma(t)} = A_i$, where $i = 1, 2, \ldots N$, the number of subsystems involved. In this paper, $D_{f\sigma(t)}$ is assumed to be of full-column rank matrix.

In order to generate the residual signal, the following switched FD filter model is used as a residual generator:

$$\begin{cases} \dot{\hat{x}}(t) = A_{\sigma'(t)}\hat{x}(t) + B_{\sigma'(t)}u(t) - L_{\sigma'(t)}(y(t) - \hat{y}(t)) \\ \hat{y}(t) = C_{\sigma'(t)}\hat{x}(t) + D_{\sigma'(t)}u(t) \\ r(t) = H_{\sigma'(t)}(y(t) - \hat{y}(t)) \end{cases} \tag{2}$$

where $L_{\sigma'(t)} \in R^{n\times m}$ and $H_{\sigma'(t)} \in R^{q\times m}$ are the parameters of the filter to be designed with respect to each subsystem $i \in \{1, 2, \ldots N\}$. Similar to the system, the switching between different modes of the filter depends on the switching signal $\sigma'(t)$, as shown in Fig. 1. If the error signal of the state estimation is $e(t) = x(t) - \hat{x}(t)$ and using the residual signal definition $r(t) = H_{\sigma'(t)}(y(t) - \hat{y}(t))$ of the filter, the residual generator dynamics are represented in the following form:

$$\begin{cases} \dot{e}(t) = (A_i + L_iC_i)e(t) + (B_{di} + L_iD_{di})d(t) \\ \qquad\quad + (B_{fi} + L_iD_{fi})f(t) \\ r(t) = H_iC_ie(t) + H_iD_{di}d(t) + H_iD_{fi}f(t). \end{cases} \tag{3}$$

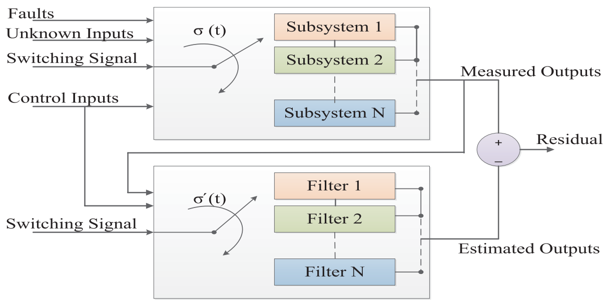

Fig. 1. Switched system and asynchronously switching FD filters.

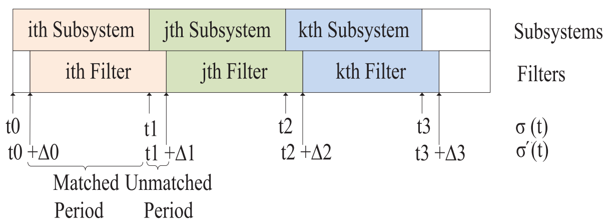

Fig. 2. Asynchronous switching between filters and subsystems.

In practice, there exists the phenomenon of asynchronous switching between the filter and the system in most of the cases. Here, in this paper, we assume that the switching sequence is unknown *a priori*; in addition, it is also assumed that a module (device) is present, which triggers the information about the activation (switching) of the subsystem. After this identification of the subsystem, the corresponding filter is switched with some delay, Δ_i. Fig. 2 shows the phenomenon of asynchronous switching. It is easy to observe that each filter lags by some time Δ_i to its corresponding subsystem. In this way, there is a matched period when ith (jth) subsystem and ith (jth) filters are in operation. On the other hand, during unmatched period, jth system but ith filters are in operation, and vice versa. These periods may cause stability problem for the overall system when switching from the matched period to the unmatched period. It is possible that $(A_i + L_iC_i)$ is stable according to (3), but during the unmatched period, these dynamics may have unstable poles. This phenomenon has been discussed in detail in Section V-B.

C. Definitions, Lemmas, and Theorems

Definition 1 (ADT [28]): For any switching signal $\sigma(t)$ and any $t_2 > t_1 > 0$, let $N_\sigma(\tau, t)$ denote the number of switchings of $\sigma(t)$ in an interval (t_1, t_2). If

$$N_\sigma(\tau, t) \leq N_o + \frac{t_2 - t_1}{\tau_\alpha}$$

holds for a given $N_o \geq 0$ and $\tau_\alpha > 0$, then the constant τ_α is called the ADT and N_o the chattering bound.

Lemma 1 (Schur's Compliment [29]): According to Schur's compliment, the following two statements are equivalent.

1) $\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{12}^T & \Phi_{22} \end{bmatrix} < 0.$

2) $\Phi_{22} < 0\ ,\ \Phi_{11} - \Phi_{12}\Phi_{22}^{-1}\Phi_{12}^T < 0.$

Theorem 1 [22]: A switched system

$$\begin{aligned} \dot{x}(t) &= A_i x(t) + B_i u(t) \\ y(t) &= C_i x(t) + D_i u(t), \quad i \in \{1, 2, \ldots N\} \end{aligned}$$

is said to be globally asymptotically stable with ADT

$$\tau_a > \tau_a{}^* = \frac{\ln(\mu 1 \mu 2)}{\zeta^*}, \quad \frac{T^-(t_0, t)}{T^+(t_0, t)} \geq \frac{\rho + \zeta^*}{\alpha - \zeta^*}, \quad 0 < \zeta^* < \alpha$$

and satisfies the H_∞ performance with index no greater than $\gamma = \max(\gamma_i)$, if there exist Lyapunov functions $V_i(x(t))$ and $V_{ij}(x(t))$ $\forall i \in \{1, 2, \ldots N\}$ such that

$$\begin{aligned} V_j(x(t)) &\leq \mu_1 V_{ij}(x(t)) \\ V_{ij}(x(t)) &\leq \mu_2 V_i(x(t)) \\ \dot{V}_i(x(t)) &\leq -\alpha V_i(x(t)) - y^T(t)y(t) + {\gamma_i}^2 u^T(t)u(t) \\ \dot{V}_{ij}(x(t)) &\leq \rho V_{ij}(x(t)) - y^T(t)y(t) + {\gamma_{ij}}^2 u^T(t)u(t) \\ &\qquad \forall i, j \in \{1, 2, \ldots N\}, \quad i \neq j. \end{aligned}$$

Note that during $[t_o, t]$, $T^-(t_0, t)$ and $T^+(t_0, t)$ denote the total matched and mismatched periods, respectively. $T^+(t_0, t)$ is equal to Δ_i.

D. H_-/H_∞ Index and Fault Isolation

Each subsystem of the considered switched system is linear time invariant, and therefore each subsystem can be individually represented by a transfer function $G_i(s) = (A_i, B_i, C_i, D_i)$ $\forall i \in \{1, 2, \ldots, N\}$. It is intended to design an FDF such that the following are present:

1) maximum effect of fault on the residual signal, that is, $\| r_f(t) \|_2 \geq \beta_i \| f(t) \|_2$ $\forall i \in \{1, 2, \ldots N\}$;
2) minimum effect of unknown inputs on the residual signal, that is, $\| r_d(t) \|_2 \leq \gamma_i \| d(t) \|_2$ $\forall i \in \{1, 2, \ldots N\}$ where r_f is the effect of fault on residual and r_d is the effect of disturbance on the residual signal.

Residual signal $R(s)$ during each mode is given as

$$R(s) = F_i(s)(G_{di}(s)d(s) + G_{fi}(s)f(s))$$

where $F_i(s) = (A_i + L_iC_i, L_i, H_iC_i, H_i)$ is the postfilter for each subsystem of the switched system, and also

$$\begin{aligned} G_i &= (A_i, B_i, C_i, D_i) \\ G_{di} &= (A_i, B_{di}, C_i, D_{di}) \\ G_{fi} &= (A_i, B_{fi}, C_i, D_{fi}). \end{aligned}$$

The transfer function $F_i(s)G_i(s)$ is given as

$$F_i(s) = (A_i + L_iC_i, B_{fi} + L_iD_{fi}, H_iC_i, H_iD_{fi})$$

for further details, see [27].

The above-stated H_-/H_∞ objective is achievable if a filter $F_i(s)$ can be designed for each mode by finding L_i and H_i such that γ_i is minimized and simultaneously ensuring $\beta_i \geq 1$ under the ADT constraint. In addition, the design of filter should be such that, to achieve the fault isolation, every fault should exclusively influence a residual signal [2].

The above-explained problem can be presented in short as follows:

$$\gamma_i = \inf \| F_i(s)G_{di}(s) \|_\infty \quad \text{s.t.} \ \| F_i(s)G_{fi}(s) \|_- \geq \beta_i$$

and $F_i(s) = (A_i + L_iC_i, L_i, H_iC_i, H_i) \in RH_\infty^{q \times m}$.

E. Assumptions

While designing the FD filter (2), the following assumptions are made.

1) The pair (A_i, C_i) is detectable, $\forall\ i \in \{1, 2 \ldots N\}$; in this way, the FDF can be designed for each mode of the system.
2) G_{fi} has no zeros on the imaginary axis, $\forall\ i \in \{1, 2, \ldots N\}$. This is the requirement so that fault at any frequency can have effect on the residual signal. The assumption also implies that D_{fi} should be of full column rank for each mode.
3) $m > q$, the number of outputs is greater than the number of faults.

After the problem has been formulated, in the next section, we discuss the solution to H_-/H_∞ and fault isolation problem.

III. Solution to the H_-/H_∞ Problem

H_-/H_∞ solution is a sort of compromise between maximizing fault sensitivity level and minimizing disturbance attenuation level. To this end, different variants may exist. In this paper, compromise between sensitivity and attenuation level is proposed such that fault isolation is also possible. We propose this by setting H_- index, β_i, and then optimize (minimize) the H_∞ norm, $\gamma_{\mathbf{i}}$. Conditions for the matched and unmatched periods are derived with details in the following theorem.

Theorem 2: Given scalars, $\alpha > 0, \rho > 0$, $\mu_1 \geq 1$, $\mu_2 \geq 1$, and $\beta_i \geq 1$, if there exist $P_i > 0$ and $P_{ij} > 0$ for $i \neq j$ and $i, j \in N$ such that the following inequalities:

$$\begin{bmatrix} P_{11ij} & P_{12ij} \\ * & P_{22ij} \end{bmatrix} > 0, \quad \begin{bmatrix} P_{11i} & P_{12i} \\ * & P_{22i} \end{bmatrix} > 0 \tag{4}$$

$$\begin{bmatrix} P_{11j} & P_{12j} \\ * & P_{22j} \end{bmatrix} \leq \mu_1 \begin{bmatrix} P_{11ij} & P_{12ij} \\ * & P_{22ij} \end{bmatrix} \tag{5}$$

$$\begin{bmatrix} P_{11ij} & P_{12ij} \\ * & P_{22ij} \end{bmatrix} \leq \mu_2 \begin{bmatrix} P_{11i} & P_{12i} \\ * & P_{22i} \end{bmatrix} \tag{6}$$

$$\begin{bmatrix} \Psi_{11} & \Psi_{12} & P_{11i}B_i + P_{12i}B_i & \Psi_{14} & \Psi_{15} \\ * & \Psi_{22} & {P_{12i}}^T B_i + P_{22i}B_i & \Psi_{24} & \Psi_{25} \\ * & * & -{\gamma_i}^2 I & 0 & 0 \\ * & * & * & -{\gamma_i}^2 I & \Psi_{45} \\ * & * & * & * & -I \end{bmatrix} < 0 \tag{7}$$

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & \Omega_{15} \\ * & \Omega_{22} & \Omega_{23} & \Omega_{24} & \Omega_{25} \\ * & * & -{\gamma_{ij}}^2 & 0 & \Omega_{35} \\ * & * & * & -{\gamma_{ij}}^2 I & \Omega_{45} \\ * & * & * & * & -I \end{bmatrix} < 0 \tag{8}$$

hold, where

$$\begin{aligned}
\Psi_{11} &= {A_i}^T P_{11i} + C_i^T {D_{fi}{}^\dagger}^T {B_{fi}}^T {P_{12i}}^T + P_{11i}A_i \\
&\quad - C_i^T {D_{fi}{}^\perp}^T {W_i}^T + P_{12i}B_{fi}{D_{fi}}^\dagger C_i \\
&\quad - W_i {D_{fi}}^\perp C_i + \alpha P_{11i} \\
\Psi_{12} &= {A_i}^T P_{12i} + P_{12i}A_i + \alpha P_{12i} - P_{12i}B_{fi}D_{fi}^\dagger C_i \\
&\quad + {C_i}^T {D_{fi}{}^\dagger}^T {B_{fi}}^T P_{22i} - C_i {D_{fi}{}^\dagger}^T {X_i}^T \\
&\quad + W_i {D_{fi}}^\perp C_i \\
\Psi_{14} &= P_{11i}B_{di} + P_{12i}B_{fi}D_{fi}^\dagger D_{di} - W_i D_{fi}^\perp D_{di} \\
\Psi_{15} &= {C_i}^T {D_{fi}{}^\dagger}^T {\beta_i}^T + {C_i}^T {D_{fi}{}^\perp}^T {S_i}^T {\beta_i}^T \\
\Psi_{22} &= {A_i}^T P_{22i} + \alpha P_{22i} - {C_i}^T {D_{fi}^\dagger}^T B_{fi}^T P_{22i} + P_{22i}A_i \\
&\quad + {C_i}^T {D_{fi}^\perp}^T {X_i}^T - P_{22i}B_{fi}{D_{fi}}^\dagger C_i + X_i {D_{fi}}^\perp C_i \\
\Psi_{24} &= {P_{12i}}^T B_{di} + P_{22i}B_{fi}{D_{fi}}^\dagger D_{di} - X_i {D_{fi}}^\perp D_{di} \\
\Psi_{25} &= -{C_i}^T {D_{fi}{}^\dagger}^T {\beta_i}^T - {C_i}^T {D_{fi}{}^\perp}^T {S_i}^T {\beta_i}^T \\
\Psi_{45} &= {D_{di}}^T {D_{fi}{}^\dagger}^T {\beta_i}^T + {D_{di}}^T {D_{fi}{}^\perp}^T {S_i}^T {\beta_i}^T \\
\Omega_{11} &= {A_j}^T P_{11ij} + C_j^T {D_{fi}^\dagger}^T {B_{fi}}^T {P_{12ij}}^T - C_j^T {D_{fi}^\dagger}^T {Y_i}^T \\
&\quad - \sigma P_{11ij} + P_{11ij}A_j + P_{12ij}B_{fi}{D_{fi}}^\dagger C_j - Y_i {D_{fi}}^\perp C_j \\
\Omega_{12} &= {A_j}^T P_{12ij} + P_{12ij}A_i - \sigma P_{12ij} - P_{12ij}B_{fi}D_{fi}^\dagger C_i \\
&\quad + Y_i {D_{fi}}^\perp C_i + C_j^T {D_{fi}{}^\dagger}^T {B_{fi}}^T P_{22ij} - C_j^T {D_{fi}{}^\perp}^T {Z_i}^T \\
\Omega_{13} &= P_{11ij}B_j + P_{12ij}B_i - P_{12ij}B_{fi}D_{fi}^\dagger D_i \\
&\quad + P_{12ij}B_{fi}{D_{fi}}^\dagger D_j - Y_i {D_{fi}}^\dagger D_j + Y_i {D_{fi}}^\perp D_i \\
\Omega_{14} &= P_{11ij}B_{dj} + P_{12ij}B_{fi}D_{fi}^\dagger D_{dj} - Y_i D_{fi}^\perp D_{dj} \\
\Omega_{15} &= {C_j}^T {D_{fi}{}^\dagger}^T {\beta_i}^T + {C_j}^T {D_{fi}{}^\perp}^T {S_i}^T {\beta_i}^T \\
\Omega_{22} &= {A_i}^T P_{22ij} - \sigma P_{22ij} - {C_i}^T {D_{fi}^\dagger}^T B_{fi}^T P_{22ij} \\
&\quad + P_{22ij}A_i + {C_i}^T {D_{fi}^\perp}^T {Z_i}^T - P_{22ij}B_{fi}{D_{fi}}^\dagger C_i \\
&\quad + Z_i {D_{fi}}^\perp C_i \\
\Omega_{23} &= {P_{22ij}}^T B_j + P_{22ij}B_i + P_{22ij}B_{fi}{D_{fi}}^\dagger D_j \\
&\quad - Z_i {D_{fi}}^\perp D_j - P_{22ij}B_{fi}{D_{fi}}^\dagger D_i + Z_i {D_{fi}}^\perp D_i \\
\Omega_{24} &= {P_{12ij}}^T B_{dj} + P_{22ij}B_{fi}{D_{fi}}^\dagger D_{dj} - Z_i {D_{fi}}^\perp D_{dj} \\
\Omega_{25} &= -{C_i}^T {D_{fi}{}^\dagger}^T {\beta_i}^T - {C_i}^T {D_{fi}{}^\perp}^T {S_i}^T {\beta_i}^T \\
\Omega_{35} &= {D_j}^T {D_{fi}{}^\dagger}^T {\beta_i}^T + {D_j}^T {D_{fi}{}^\perp}^T {S_i}^T {\beta_i}^T \\
&\quad - {D_i}^T {D_{fi}{}^\dagger}^T {\beta_i}^T - {D_i}^T {D_{fi}{}^\perp}^T {S_i}^T {\beta_i}^T \\
\Omega_{45} &= {D_{dj}}^T {D_{fi}{}^\dagger}^T {\beta_i}^T + {D_{dj}}^T {D_{fi}{}^\perp}^T {S_i}^T {\beta_i}^T
\end{aligned}$$

$W_i = P_{12i}R_i, X_i = P_{22i}R_i, Y_i = P_{12ij}R_i, Z_i = P_{22ij}R_i$ and

$$\begin{bmatrix} {D_{fi}}^\dagger \\ {D_{fi}}^\perp \end{bmatrix}, D_{fi} = \begin{bmatrix} I_q \\ 0 \end{bmatrix}, \text{rank}\left(\begin{bmatrix} {D_{fi}}^\dagger \\ {D_{fi}}^\perp \end{bmatrix}\right) = m.$$

Then, switched system (1) and FD filter (2) are globally asymptotically stable in augmented form, the H_-/H_∞ filter design objective is met, and occurring faults are detected and isolated for any switching signal with ADT $\tau_a > {\tau_a}^* = (\ln(\mu 1 \mu 2)/\zeta^*)$, and postfiler can be obtained by $F_i(s) = (A_i + L_iC_i, L_i, H_iC_i, H_i) \in RH_\infty^{q \times m}$. And parameters of FD filter are given by

$$\begin{aligned} L_i &= -B_{fi}{D_{fi}}^\dagger + R_i {D_{fi}}^\perp \\ H_i &= \beta_i\left({D_{fi}}^\dagger + S_i {D_{fi}}^\perp\right) \end{aligned}$$

where $R_i \in R^{n \times (m-q)}$ and $S_i \in R^{q \times (m-q)}$ are additional variables that are introduced to provide more degrees of freedom for L_i and H_i.

Proof: As it is assumed that Df_i has full column rank $\forall i \in \{1, 2, \ldots N\}$, therefore we can say that there exist matrices $D_{fi}{}^{\dagger} \in R^{(q\times m)}$ and $D_{fi}{}^{\perp} \in R^{(m-q)\times m}$, such that the following condition is satisfied:

$$\begin{bmatrix} D_{fi}{}^{\dagger} \\ D_{fi}{}^{\perp} \end{bmatrix} D_{fi} = \begin{bmatrix} I_q \\ 0 \end{bmatrix}, \quad \text{rank}\left(\begin{bmatrix} D_{fi}{}^{\dagger} \\ D_{fi}{}^{\perp} \end{bmatrix}\right) = m. \quad (9)$$

Fault Isolation: We know that

$$F_i(s)G_{fi}(s) = (A_i + L_iC_i, B_{fi} + L_iD_{fi}, H_iC_i, H_iD_{fi}) \in RH_{\infty}^{q\times m}$$

and to achieve the fault isolation capability, we require that

$$\|F_iG_{fi}\|_- \geq \beta_i \geq 1 \quad \forall i \in \{1, 2, \ldots N\}.$$

For this, $F_iG_{fi} = \beta_i I$, $\forall i \in \{1, 2, \ldots N\}$, which can be obtained easily by setting $B_{fi} + L_iD_{fi} = 0$ and $H_iD_{fi} = \beta_i I_q$ in $F_i(s)G_{fi}(s)$. Next, from these two set equations, we can find

$$\begin{aligned} L_i &= -B_{fi}D_{fi}{}^{\dagger} + R_iD_{fi}{}^{\perp} \\ H_i &= \beta_i\left(D_{fi}{}^{\dagger} + S_iD_{fi}{}^{\perp}\right) \end{aligned} \quad (10)$$

where $R_i \in R^{n\times(m-q)}$ and $S_i \in R^{q\times(m-q)}$ are additional design freedom variables that are to be found next in the H_∞ framework.

By the aforementioned approach, with the setting of terms, the fault isolation framework is achieved. For further details of the fault isolation design approach, one can see [27] for dynamical systems and [30] for switched systems under the synchronous case, while in this paper, the asynchronous case of switched systems is considered.

Now, for the desired L_i and H_i, the only remaining part of the problem is to find out the H_∞ norm of $F_iG_{di}, \forall i \in \{1, 2, \ldots N\}$. To this end, we use Theorem 1 under asynchronous paradigm during the matched period as follows.

A. Matched Period

During the matched period, ith subsystem and ith filter are in operation (Fig. 2). We augment the switched system (1) and FD filter (2) into the following compact representation:

$$\begin{cases} \dot{\tilde{x}}(t) = \tilde{A}_i\tilde{x}(t) + \tilde{B}_i\omega(t) \\ r(t) = \tilde{C}_i\tilde{x}(t) + \tilde{D}_i\omega(t) \end{cases} \quad (11)$$

where

$$\tilde{x}(t) = \begin{bmatrix} x(t)^T & \hat{x}(t)^T \end{bmatrix}^T, \quad \omega(t) = \begin{bmatrix} u(t)^T & d(t)^T \end{bmatrix}^T$$

and

$$\begin{cases} \tilde{A}_i = \begin{bmatrix} A_i & 0 \\ -L_iC_i & A_i + L_iC_i \end{bmatrix}, \quad \tilde{B}_i = \begin{bmatrix} B_i & B_{di} \\ B_i & -L_iD_{di} \end{bmatrix} \\ \tilde{C}_i = \begin{bmatrix} H_iC_i & -H_iC_i \end{bmatrix}, \quad \tilde{D}_i = \begin{bmatrix} 0 & H_iD_{di} \end{bmatrix}. \end{cases} \quad (12)$$

Using Theorem 1, during the matched period

$$\dot{V}_i(\tilde{x}(t)) \leq -\alpha V_i(\tilde{x}(t)) - r(t)^T r(t) + \gamma_i^2 \omega(t)^T \omega(t). \quad (13)$$

Considering the following Lyapunov function during the matched time:

$$V_i(\tilde{x}(t)) = \tilde{x}^T(t)P_i\tilde{x}(t). \quad (14)$$

Differentiating (14), along the trajectory of (11)

$$\dot{V}_i(\tilde{x}(t)) = \dot{\tilde{x}}^T(t)P_i\tilde{x}(t) + \tilde{x}^T(t)P_i\dot{\tilde{x}}(t). \quad (15)$$

By substituting (14) and (15), and $r(t)$ from (11) in (13), the following inequality is obtained:

$$\begin{aligned} &\dot{\tilde{x}}^T(t)P_i\tilde{x}(t) + \tilde{x}^T(t)P_i\dot{\tilde{x}}(t) + \left[\tilde{C}_i\tilde{x}(t) + \tilde{D}_i\omega(t)\right]^T \\ &\quad \times \left[\tilde{C}_i\tilde{x}(t) + \tilde{D}_i\omega(t)\right] \\ &\quad \leq -\alpha\tilde{x}^T(t)P_i\tilde{x}(t) + \gamma_i^2\omega(t)^T\omega(t). \end{aligned} \quad (16)$$

Furthermore, after substituting the expression for $\dot{\tilde{x}}(t)$ from (11) in (16)

$$\begin{aligned} &\left[\tilde{A}_i\tilde{x}(t) + \tilde{B}_i\omega(t)\right]^T P_i\tilde{x}(t) + \tilde{x}^T(t)P_i\left[\tilde{A}_i\tilde{x}(t) + \tilde{B}_i\omega(t)\right] \\ &\quad + \left(\tilde{x}^T(t)\tilde{C}_i^T + \omega^T(t)\tilde{D}_i^T\right)\left(\tilde{C}_i\tilde{x}(t) + \tilde{D}_i\omega(t)\right) \\ &\quad \leq -\alpha\tilde{x}^T(t)P_i\tilde{x}(t) + \gamma_i^2\omega(t)^T\omega(t) \end{aligned} \quad (17)$$

the above equation can be easily written in the following form:

$$\begin{aligned} &\left[\tilde{x}^T(t)\tilde{A}_i^TP_i + \omega^T(t)\tilde{B}_i^TP_i\right]\tilde{x}(t) + \tilde{x}^T(t)P_i\tilde{A}_i\tilde{x}(t) \\ &\quad + \tilde{x}^T(t)P_i\tilde{B}_i\omega(t) + \tilde{x}^T(t)\tilde{C}_i^T\tilde{C}_i\tilde{x}(t) \\ &\quad + \tilde{x}^T(t)\tilde{C}_i^T\tilde{D}_i\omega(t) + \omega^T(t)\tilde{D}_i^T\tilde{C}_i\tilde{x}(t) \\ &\quad + \omega^T(t)\tilde{D}_i^T\tilde{D}_i\omega(t) + \alpha\tilde{x}^T(t)P_i\tilde{x}(t) - \gamma_i{}^2\omega^T(t)\omega(t) \leq 0. \end{aligned} \quad (18)$$

Furthermore, the above inequality can be written as

$$\begin{bmatrix} \tilde{x}^T(t) & \omega^T(t) \end{bmatrix} M \begin{bmatrix} \tilde{x}(t) \\ \omega(t) \end{bmatrix} \leq 0 \quad (19)$$

where

$$M = \begin{bmatrix} \tilde{A}_i{}^TP_i + P_i\tilde{A}_i + \tilde{C}_i{}^T\tilde{C}_i + \alpha P_i & P_i\tilde{B}_i + \tilde{C}_i{}^T\tilde{D}_i \\ * & \tilde{D}_i{}^T\tilde{D}_i - \gamma_i^2 I \end{bmatrix}.$$

For (19) to hold, it is required that

$$M < 0. \quad (20)$$

After Schur's compliment is applied to (20), we get

$$\begin{bmatrix} \tilde{A}_i{}^TP_i + P_i\tilde{A}_i + \alpha P_i & P_i\tilde{B}_i & \tilde{C}_i{}^T \\ * & -\gamma_i^2 I & \tilde{D}_i{}^T \\ * & * & -I \end{bmatrix} < 0. \quad (21)$$

Next, we substitute the expressions of L_i and H_i into (21) and get the following LMI:

$$\begin{bmatrix} \Psi_{11} & \Psi_{12} & P_{11i}B_i + P_{12i}B_i & \Psi_{14} & \Psi_{15} \\ * & \Psi_{22} & P_{12i}{}^TB_i + P_{22i}B_i & \Psi_{24} & \Psi_{25} \\ * & * & -\gamma_i{}^2 I & 0 & 0 \\ * & * & * & -\gamma_i{}^2 I & \Psi_{45} \\ * & * & * & * & -I \end{bmatrix} < 0 \quad (22)$$

where

$$\Psi_{11} = {A_i}^T P_{11i} + C_i^T {D_{fi}^{\perp}}^T {B_{fi}}^T {P_{12i}}^T + P_{11i} A_i - C_i^T {D_{fi}^{\perp}}^T {R_i}^T {P_{12i}}^T + P_{12i} B_{fi} {D_{fi}}^{\dagger} C_i - P_{12i} R_i {D_{fi}}^{\perp} C_i + \alpha P_{11i}$$

$$\Psi_{12} = {A_i}^T P_{12i} + P_{12i} A_i + \alpha P_{12i} - P_{12i} B_{fi} D_{fi}^{\dagger} C_i + {C_i}^T {D_{fi}^{\dagger}}^T {B_{fi}}^T P_{22i} - C_i {D_{fi}^{\dagger}}^T {X_i}^T + P_{12i} R_i {D_{fi}}^{\perp} C_i$$

$$\Psi_{14} = P_{11i} B_{di} + P_{12i} B_{fi} D_{fi}^{\dagger} D_{di} - P_{12i} R_i D_{fi}^{\perp} D_{di}$$

$$\Psi_{15} = {C_i}^T {D_{fi}^{\dagger}}^T {\beta_i}^T + {C_i}^T {D_{fi}^{\perp}}^T {S_i}^T {\beta_i}^T$$

$$\Psi_{22} = {A_i}^T P_{22i} + \alpha P_{22i} - {C_i}^T {D_{fi}^{\dagger}}^T B_{fi}^T P_{22i} + P_{22i} A_i + {C_i}^T {D_{fi}^{\perp}}^T {R_i}^T P_{22i} - P_{22i} B_{fi} {D_{fi}}^{\dagger} C_i + P_{22i} R_i {D_{fi}}^{\perp} C_i$$

$$\Psi_{24} = {P_{12i}}^T B_{di} + P_{22i} B_{fi} {D_{fi}}^{\dagger} D_{di} - P_{22i} R_i {D_{fi}}^{\perp} D_{di}$$

$$\Psi_{25} = -{C_i}^T {D_{fi}^{\dagger}}^T {\beta_i}^T - {C_i}^T {D_{fi}^{\perp}}^T {S_i}^T {\beta_i}^T$$

$$\Psi_{45} = {D_{di}}^T {D_{fi}^{\dagger}}^T {\beta_i}^T + {D_{di}}^T {D_{fi}^{\perp}}^T {S_i}^T {\beta_i}^T.$$

Inequality (22) is a bilinear matrix inequality (BMI). In order to transform the BMI into an LMI, we use the following substitutions:

$$P_{12i} R_i = W_i \text{ and } P_{22i} R_i = X_i.$$

Thus, (7) for the matched period is derived. Next, we drive the conditions for the unmatched period.

B. Unmatched Period

During the unmatched period, jth subsystem and ith filter are in operation (Fig. 2). We augment the switched system (1) and FD filter (2) as follows:

$$\begin{cases} \dot{\tilde{x}}(t) = \tilde{A}_{ij} \tilde{x}(t) + \tilde{B}_{ij} \omega(t) \\ r(t) = \tilde{C}_{ij} \tilde{x}(t) + \tilde{D}_{ij} \omega(t) \end{cases} \quad (23)$$

where $\tilde{x}(t) = \left[x(t)^T \;\; \hat{x}(t)^T\right]^T$, $\omega(t) = \left[u(t)^T \;\; d(t)^T\right]^T$, and

$$\begin{cases} \tilde{A}_{ij} = \begin{bmatrix} A_j & 0 \\ -L_i C_j & A_i + L_i C_i \end{bmatrix} \\ \tilde{B}_{ij} = \begin{bmatrix} B_j & B_{dj} \\ B_i - L_i D_j + L_i D_i & -L_i D_{dj} \end{bmatrix} \\ \tilde{C}_{ij} = [H_i C_j \quad -H_i C_i] \\ \tilde{D}_{ij} = [H_i D_j - H_i D_i \quad H_i D_{dj}]. \end{cases} \quad (24)$$

Using Theorem 1, during the unmatched period

$$\dot{V}_{ij}(x(t)) \le \rho V_{ij}(x(t)) - y^T(t) y(t) + {\gamma_{ij}}^2 u^T(t) u(t). \quad (25)$$

Considering the following Lyapunov function during the unmatched time:

$$V_{ij}(\tilde{x}(t)) = \tilde{x}^T(t) P_{ij} \tilde{x}(t). \quad (26)$$

Differentiating (26), along the trajectory of (23)

$$\dot{V}_{ij}(\tilde{x}(t)) = \dot{\tilde{x}}^T(t) P_{ij} \tilde{x}(t) + \tilde{x}^T(t) P_{ij} \dot{\tilde{x}}(t). \quad (27)$$

By substituting (26) and (27), and $r(t)$ from (23) into (25)

$$\dot{\tilde{x}}^T(t) P_{ij} \tilde{x}(t) + \tilde{x}^T(t) P_{ij} \dot{\tilde{x}}(t) + \left[\tilde{C}_{ij} \tilde{x}(t) + \tilde{D}_{ij} \omega(t)\right]^T \times \left[\tilde{C}_{ij} \tilde{x}(t) + \tilde{D}_{ij} \omega(t)\right] \le \rho \; \tilde{x}^T(t) P_{ij} \tilde{x}(t) + \gamma_{ij}^2 \omega(t)^T \omega(t). \quad (28)$$

Furthermore, after substituting the expression for $\dot{\tilde{x}}(t)$ from (23) into (28)

$$\left[\tilde{A}_{ij} \tilde{x}(t) + \tilde{B}_{ij} \omega(t)\right]^T P_{ij} \tilde{x}(t) + \tilde{x}^T(t) P_{ij} \tilde{A}_{ij} \tilde{x}(t) + \left(\tilde{x}^T(t) \tilde{C}_{ij}^T + \omega^T(t) \tilde{D}_{ij}^T\right)\left(\tilde{C}_{ij} \tilde{x}(t) + \tilde{D}_{ij} \omega(t)\right) + \tilde{x}^T(t) P_{ij} \tilde{B}_{ij} \omega(t) \le \rho \; \tilde{x}^T(t) P_{ij} \tilde{x}(t) + \gamma_{ij}^2 \omega(t)^T \omega(t) \quad (29)$$

the above equation can be easily written in the following form:

$$\left[\tilde{x}^T(t) \tilde{A}_{ij}^T P_i + \omega^T(t) \tilde{B}_{ij}^T P_i\right] \tilde{x}(t) + \tilde{x}^T(t) P_{ij} \tilde{A}_{ij} \tilde{x}(t) + \tilde{x}^T(t) P_{ij} \tilde{B}_{ij} \omega(t) + \tilde{x}^T(t) \tilde{C}_{ij}^T \tilde{C}_{ij} \tilde{x}(t) + \tilde{x}^T(t) \tilde{C}_{ij}^T \tilde{D}_{ij} \omega(t) + \omega^T(t) \tilde{D}_{ij}^T \tilde{C}_{ij} \tilde{x}(t) + \omega^T(t) \tilde{D}_{ij}^T \tilde{D}_{ij} \omega(t) - \rho \; \tilde{x}^T(t) P_{ij} \tilde{x}(t) - {\gamma_{ij}}^2 \omega^T(t) \omega(t) \le 0. \quad (30)$$

Furthermore, (30) can be written as

$$\left[\tilde{x}^T(t) \quad \omega^T(t)\right] M \begin{bmatrix} \tilde{x}(t) \\ \omega(t) \end{bmatrix} \le 0 \quad (31)$$

where M is

$$\begin{bmatrix} {\tilde{A}_{ij}}^T P_{ij} + P_{ij} \tilde{A}_{ij} + {\tilde{C}_{ij}}^T \tilde{C}_{ij} - \rho P_{ij} & P_{ij} \tilde{B}_{ij} + {\tilde{C}_{ij}}^T \tilde{D}_{ij} \\ * & {\tilde{D}_{ij}}^T \tilde{D}_{ij} - \gamma_{ij}^2 I \end{bmatrix}.$$

To hold (31), it is required that

$$M < 0. \quad (32)$$

After Schuar's compliment is applied to (32), we get

$$\begin{bmatrix} {\tilde{A}_{ij}}^T P_{ij} + P_{ij} \tilde{A}_{ij} - \rho P_{ij} & P_{ij} \tilde{B}_{ij} & {\tilde{C}_{ij}}^T \\ * & -\gamma_{ij}^2 I & {\tilde{D}_{ij}}^T \\ * & * & -I \end{bmatrix} < 0. \quad (33)$$

Next, we substitute the expressions of L_i and H_i from (10) into (33) and get the following inequality:

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & \Omega_{15} \\ * & \Omega_{22} & \Omega_{23} & \Omega_{24} & \Omega_{25} \\ * & * & -{\gamma_{ij}}^2 I & 0 & \Omega_{35} \\ * & * & * & -{\gamma_{ij}}^2 I & \Omega_{45} \\ * & * & * & * & -I \end{bmatrix} < 0 \quad (34)$$

where

$$\Omega_{11} = {A_j}^T P_{11ij} + C_j^T {D_{fi}^{\dagger}}^T {B_{fi}}^T {P_{12ij}}^T + P_{11ij} A_j - C_j^T {D_{fi}^{\perp}}^T {R_i}^T {P_{12ij}}^T + P_{12ij} B_{fi} {D_{fi}}^{\dagger} C_j - P_{12i} R_i {D_{fi}}^{\perp} C_i - \rho P_{11ij}$$

$$\Omega_{12} = {A_j}^T P_{12ij} + P_{12ij} A_i - P_{12i} B_{fi} D_{fi}^{\dagger} C_i + {C_j}^T {D_{fi}^{\dagger}}^T {B_{fi}}^T P_{22ij} - C_j {D_{fi}^{\dagger}}^T {R_i}^T P_{22ij} + P_{12i} R_i {D_{fi}}^{\perp} C_i - \rho P_{12ij}$$

$$\begin{aligned}
\Omega_{13} &= P_{11ij}B_j + P_{12ij}B_i - P_{12ij}B_{fi}D_{fi}^{\dagger}D_i \\
&\quad + P_{12ij}B_{fi}{D_{fi}}^{\dagger}D_j - P_{12ij}R_i{D_{fi}}^{\dagger}D_j \\
&\quad + P_{12ij}R_i{D_{fi}}^{\perp}D_i \\
\Omega_{14} &= P_{12ij}B_{fi}D_{fi}^{\dagger}D_{dj} - P_{12ij}R_iD_{fi}^{\perp}D_{dj} \\
&\quad + P_{11ij}B_{dj} \\
\Omega_{15} &= {C_j}^T{{D_{fi}}^{\dagger}}^T{\beta_i}^T + {C_j}^T{{D_{fi}}^{\perp}}^T{S_i}^T{\beta_i}^T \\
\Omega_{22} &= {A_i}^TP_{22ij} - \rho P_{22ij} - {C_i}^T{D_{fi}^{\dagger}}^TB_{fi}^TP_{22ij} \\
&\quad + P_{22ij}A_i + {C_i}^T{D_{fi}^{\perp}}^T{R_i}^TP_{22ij} \\
&\quad - P_{22ij}B_{fi}{D_{fi}}^{\dagger}C_i + P_{22ij}R_i{D_{fi}}^{\perp}C_i \\
\Omega_{23} &= {P_{12ij}}^TB_j + P_{22ij}B_i - P_{22ij}B_{fi}{D_{fi}}^{\dagger}D_i \\
&\quad + P_{22ij}R_i{D_{fi}}^{\perp}D_i + P_{22ij}B_{fi}{D_{fi}}^{\dagger}D_j \\
&\quad - P_{22ij}R_i{D_{fi}}^{\perp}D_j \\
\Omega_{24} &= {P_{12ij}}^TB_{dj} + P_{22ij}B_{fi}{D_{fi}}^{\dagger}D_{dj} \\
&\quad - P_{22ij}R_i{D_{fi}}^{\perp}D_{dj} \\
\Omega_{25} &= -{C_i}^T{{D_{fi}}^{\dagger}}^T{\beta_i}^T - {C_i}^T{{D_{fi}}^{\perp}}^T{S_i}^T{\beta_i}^T \\
\Omega_{35} &= {D_j}^T{{D_{fi}}^{\dagger}}^T{\beta_i}^T + {D_j}^T{{D_{fi}}^{\perp}}^T{S_i}^T{\beta_i}^T \\
&\quad - {D_i}^T{{D_{fi}}^{\dagger}}^T{\beta_i}^T - {D_i}^T{{D_{fi}}^{\perp}}^T{S_i}^T{\beta_i}^T \\
\Omega_{45} &= {D_{dj}}^T{{D_{fi}}^{\dagger}}^T{\beta_i}^T + {D_{dj}}^T{{D_{fi}}^{\perp}}^T{S_i}^T{\beta_i}^T.
\end{aligned}$$

Inequality (34) is a BMI. In order to transform the BMI into an LMI, we use the following substitutions:

$$P_{12ij}R_i = Y_i \text{ and } P_{22ij}R_i = Z_i.$$

Thus, (8) is derived. In order to satisfy (7) and (8), L_i and H_i must be computed according to (10). Note that the feasibility of (7) and (8) together with (10) requires that D_{fi} is full column rank.

Now, the proof is completed, which ensures the fault sensitivity level to be $\beta_i, \forall i \in \{1, 2, \ldots N\}$, disturbance attenuation level $\gamma_i, \forall i \in \{1, 2, \ldots N\}$, and fault isolation capability achieved by the derived filter. ■

Remark 1: In the FD framework, the residual signal should be generated such that it contains the information of faults only. Due to this reason, effects of all other signals, that is, $(u(t), d(t))$ on the residual signal, need to be minimized/eliminated. To achieve this purpose, $u(t)$ is included with $d(t)$ in $\omega(t)$ vector. However, for control purpose, $u(t)$ can be dealt separately. Moreover, control inputs $u(t)$, in general, are bounded in control loops. The bounds of $u(t)$ depend on a particular application. Here, the only reason to assume norm-bounded $u(t)$ is to study the effect of control input $(u(t))$ and disturbance $(d(t))$ on residual signal $(r(t))$ in unified way.

In the next section, we present the algorithm in stepwise simplified form to design our objective filter, which is based on the results derived in Theorems 2 and 3.

C. Algorithm

Let the model of the switched system is given as in (1) along with the assumptions as in II-E.

1) Check the detectability of all subsystems (modes), i.e., if (A_i, C_i) is detectable $\forall i \in \{1, 2, \ldots N\}$, if yes then proceed to the next step.
2) Find $D_{fi}^{\dagger}$ and $D_{fi}^{\perp}$ such that
$$\begin{bmatrix} {D_{fi}}^{\dagger} \\ {D_{fi}}^{\perp} \end{bmatrix} D_{fi} = \begin{bmatrix} I_{nf} \\ 0 \end{bmatrix}, \text{rank}\left(\begin{bmatrix} {D_{fi}}^{\dagger} \\ {D_{fi}}^{\perp} \end{bmatrix}\right) = m.$$
3) Set the ADT parameters, $\mu_1, \mu_2, \alpha, \rho$, and $\beta_i = 1 \ \forall \ i \in \{1, 2, \ldots N\}$, and then solve the (4)–(8) simultaneously to get the optimal values of γ_i, $\forall \ i \in \{1, 2, \ldots N\}$.
4) Find variables R_i and $S_i \ \forall \ i \in \{1, 2, \ldots N\}$ from Step 3 and set the filter parameters as $L_i = -B_{fi}{D_{fi}}^{\dagger} + R_i{D_{fi}}^{\perp}$, and $H_i = \beta_i({D_{fi}}^{\dagger} + S_iD_{fi}^{\perp})$.

IV. Threshold Computation and Residual Evaluation

Residual evaluation is an important step after the residual generation, since the residual may not be zero even if there is no fault in the system. In the literature, there exist many variants of residual evaluation function, for instance, linear systems [31], nonlinear systems [32], and switched systems [16]. In this paper, we use the evaluation function based on root-mean-square (rms) energy of the residual signal that is given as

$$J_{\text{RMS}} = \| r(t) \|_{\text{RMS}} = \left(\frac{1}{T}\int_t^{t+T} \| r(\tau) \|^2\right)^{\frac{1}{2}} d\tau \tag{35}$$

where T is the evaluation window.

Along with the evaluation function, we also need to compute the threshold value for efficient detection of faults. Threshold value is the maximum influence of unknown inputs (disturbances, noises) and model uncertainties on the residual signal in the absence of faults. Threshold can either be fixed or adaptive [11], [16], [31], [32]. In this paper, the following threshold is employed, which is designed simply when the fault is zero:

$$J_{\text{th,RMS}} = \sup_{d(t)\in l_2, f(t)=0} J_{\text{RMS}}. \tag{36}$$

Then, after finding the evaluation function and computing threshold setting, decision about the presence of fault in the system is made by the following logic:

1) $J_{\text{RMS}} < J_{\text{th,RMS}} \Longrightarrow$ No Fault;
2) $J_{\text{RMS}} > J_{\text{th,RMS}} \Longrightarrow$ Fault Detected.

V. Application of Designed Filters

In this section, the proposed framework is utilized for FDI in the case studies of highly maneuverable aircraft technology (HiMAT) and boost–buck converter. The simulation platform for both cases is the same for a time of 30 s. Subsystem 1 is activated when $\sigma(t) = 1$ and subsystem 2 when $\sigma(t) = 0$. In the case of HiMAT, one fault $f(t) = 0.5$ is applied, while in the case of boost–buck converter, two faults, $f_1(t) = 1$ and $f_2(t) = 0.5$, are simulated. The details of switching behavior of the corresponding subsystems, filters, and faults are shown in Figs. 3 and 7 for both cases. Further, in both case studies, we choose fault sensitivity level, as $\beta_i = 1 \forall \ i \in \{1, 2\}$, and other parameter values, as $\mu_1 = \mu_2 = 1.5$ and $\alpha = 0.5$,

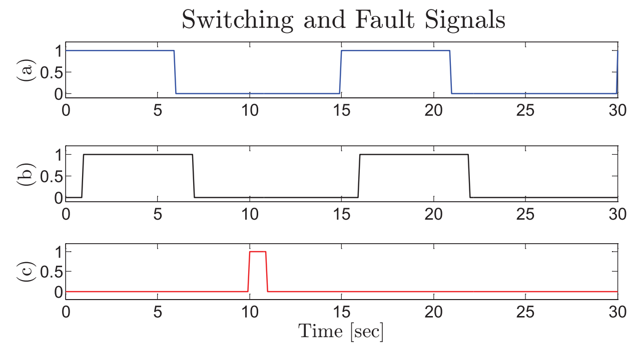

Fig. 3. (a) Switching signal $\sigma(t)$ for subsystems. (b) Switching signal $\sigma'(t)$ for filters. (c) Fault signal $f(t)$.

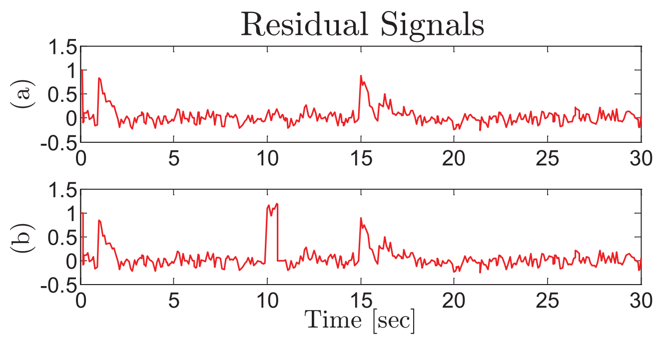

Fig. 4. (a) Residual signal without fault. (b) Residual signal with fault.

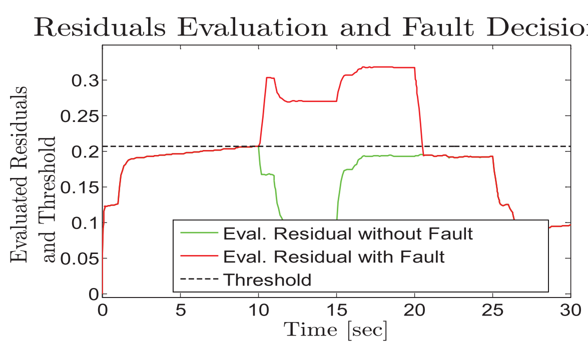

Fig. 5. Evaluated residual signals and threshold.

$\rho = 0.5$. To this end, switching signal $\sigma(t)$ is applied with an ADT value of 1.6218, that is, the switching interval between any two subsystems is greater than 1.6218. Furthermore, for simulation study, we take the disturbance signals as L_2 norm bounded with $\delta_{d,2} \leq 1$ for each mode. Next, we discuss the results of both cases.

A. Case Study 1: HiMAT Vehicle

First, we consider the model of a HiMAT [33], [34], which can be considered as a 2-D switched system, where $x_1(t)$ and $x_2(t)$ are the angle of attack and pitch rate, respectively. The dynamics of two mode switched systems are given in the Appendix. Using Theorem 2, the parameters of the filter are obtained, which are also given in the Appendix. We find disturbance attenuation levels $\gamma_1 = 0.1473$ and $\gamma_2 = 0.1367$ from LMI solution. Moreover, the equations $B_{fi} + L_i D_i = 0$ and $H_i D_i = \beta_i I_q$ are also satisfied.

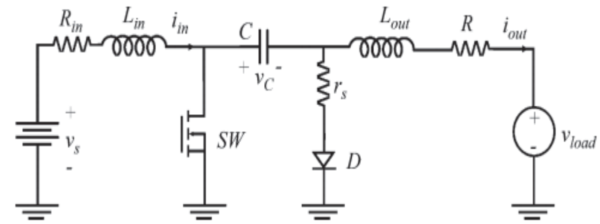

Fig. 6. Boost–buck converter circuit diagram [35].

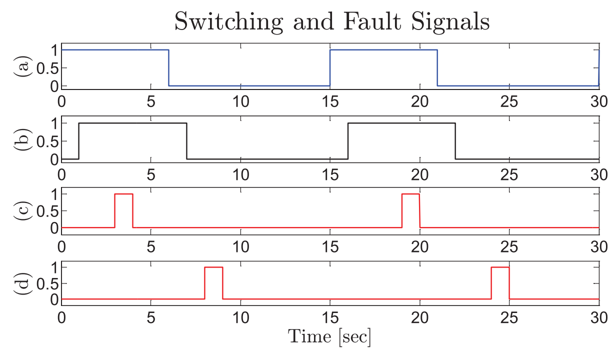

Fig. 7. (a) Switching signal $\sigma(t)$ for subsystems. (b) Switching signal $\sigma'(t)$ for filters. (c) Fault signal $f_1(t)$. (d) Fault signal $f_2(t)$.

Then, fault is simulated as in Fig. 3. Under the above-mentioned setting, the residual signals for the system in the absence and presence of faults are shown in Fig. 4. It can be observed that the residual signal is not zero even when there is no fault present in the system. To this end, proper FD is not possible, and false alarms may be generated. That is why, residual signals need to be evaluated, and then a threshold level has to be set for detecting the faults. Using (24), the threshold value is set to be 0.2072. Evaluated residuals and thresholds have been plotted in Fig. 5, whereas a window size of $T = 10$ is used for evaluating the residual signal. It is easy to observe that the fault is detected effectively in a very short span of time, when the evaluated residual signal crosses the threshold level.

Remark 2: In the discussed case of HiMAT system, we introduced only one fault, according to our assumption that the number of faults should be less than outputs. Therefore, fault isolation is not applicable in this case. To demonstrate the fault isolation along with FD successfully, we consider next the example of boost–buck converter with the case of occurrence of two faults.

B. Case Study 2: Boost–Buck Converter

In this section, we consider the boost–buck converter switched system [35], [36] for simulating the proposed design technique; the circuit is shown in Fig. 6, and the dynamic model of this system in two switching modes is given as

$$A_1 = \begin{bmatrix} -\dfrac{R_{\text{in}} + r_s}{L_{\text{in}}} & \dfrac{r_s}{L_{\text{in}}} & -\dfrac{1}{L_{\text{in}}} \\ \dfrac{r_s}{L_{\text{out}}} & -\dfrac{R + r_s}{L_{\text{out}}} & 0 \\ \dfrac{1}{C} & 0 & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} \dfrac{1}{L_{\text{in}}} \\ 0 \\ 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -\frac{R_{\text{in}}}{L_{\text{in}}} & 0 & 0 \\ 0 & -\frac{R}{L_{\text{out}}} & -\frac{1}{L_{\text{out}}} \\ 0 & \frac{1}{C} & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ -\frac{1}{L_{\text{out}}} \\ 0 \end{bmatrix}$$

where $x = [i_{\text{in}} \; i_{\text{out}} \; v_c]$ is the state vector of the converter, and we assume that measurements of all state variables are available. The parameter values of boost–buck converter are $R_{\text{in}} = 30\ \Omega$, $L_{\text{in}} = 20$ mH, $C = 20\ \mu$F, $r_s = 10\ \Omega$, $L_{\text{out}} = 20$ mH, $R = 30\ \Omega$, and $v_s = 15$ V. According to these values, system matrices are shown in the Appendix.

If we use the following filter parameters:

$$L_1 = \begin{bmatrix} -0.3245 & -100.5023 & 0 \\ 1.4569 & -1.2563 & 100 \\ 10 & 0 & 1000.6584 \end{bmatrix}$$

$$H_1 = \begin{bmatrix} 2.2388 & 7.72546 \\ -10.2546 & 8.2564 \\ -11.2546 & 10.2365 \end{bmatrix}^T$$

we can find that eigenvalues of the overall system are stable when subsystem and filter both are in mode 1 (matched period), that is, $A_1 + L_1C_1$ has stable eigenvalues. However, when the subsystem switches to mode 2 and the filter is still in mode 1 (unmatched period), then eigenvalues of $A_2 + L_1C_2$ become unstable.

Whereas according to our proposed design, by solving (4)–(8) of Theorem 2, we find the filter parameters (given in the Appendix), and in this case, the disturbance attenuation levels are $\gamma_1 = 0.1726$ and $\gamma_2 = 0.1446$. It is easy to observe that, according to (3), the eigenvalues of $A_i + L_iC_i$ are stable during the matched period with the following possible cases.

Case 1: If subsystem and filter are in mode 1, then $A_1 + L_1C_1$ has stable eigenvalues.

Case 2: If subsystem and filter are in mode 2, then $A_2 + L_2C_2$ has stable eigenvalues.

Similarly, during the unmatched period, the eigenvalues of $A_i + L_iC_i$ are stable in the following possible cases.

Case 1: If subsystem has switched to mode 2 while filter is still in mode 1, then $A_2 + L_1C_2$ has stable eigenvalues.

Case 2: If subsystem has switched to mode 1 while filter is still in mode 2, then $A_1 + L_2C_1$ has stable eigenvalues.

Now, it is worth noting that if the design of the filter does not consider the unmatched interval of the asynchronous case and treats the only matched time cases, then the system may get unstable while switching from the matched period to unmatched period as discussed just before.

Then, we simulate the systems as in Fig. 7. The residual signals for the system in the absence and presence of faults are shown in Fig. 8. Here, it is easy to observe that $f_1(t)$ affects only the residual 1, whereas $f_2(t)$ exclusively affects residual 2. In this way, both faults are not only successfully detected but also isolated (located). It is worth noting that two design requirements for fault isolation, $B_{fi} + L_iD_i = 0$ and

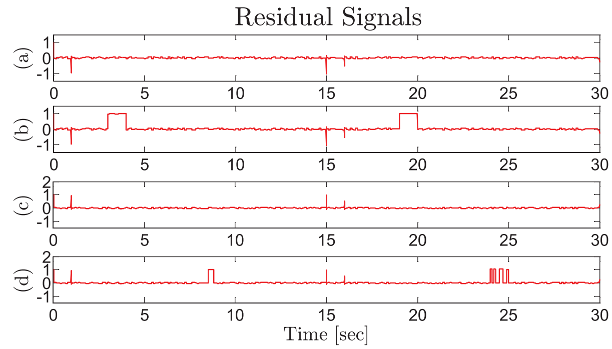

Fig. 8. (a) Residual signal 1 without fault. (b) Residual signal 1 with fault $f_1(t)$. (c) Residual signal 2 without fault. (d) Residual signal 2 with fault $f_2(t)$.

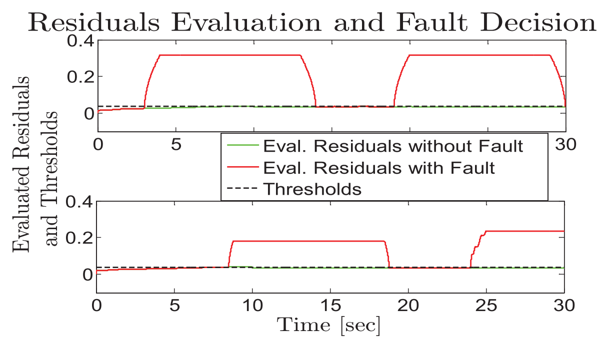

Fig. 9. Evaluated residual signals and thresholds.

$H_iD_i = \beta_iI_q$, are also satisfied. Finally, to avoid false alarms, residuals are evaluated by rms, and thresholds are set according to 24, which are shown in Fig. 9. For residual evaluation, the window size T is set to 10, while thresholds for residual signal 1 and residual signal 2 are 0.0390 and 0.0410, respectively, in this case. Fault is detected properly when residual evaluated signals 1 and 2 cross their respective threshold values.

VI. Conclusion

In this paper, the problem of FDI for continuous-time switched control systems under asynchronous switching has been investigated, and the solution is provided in the form of a mixed H_-/H_∞ filtering approach using model-based FD framework. Average dwell time constraint has been considered while employing piecewise Lyapunov function. The results are derived in the form of LMIs. The advantage of the proposed solution is that not only FD but also fault isolation is possible, which has been demonstrated by the design of case studies, HiMAT and boost–buck power converter. The results instigate the authors to expand the proposed results for fault estimation and FTC design in asynchronous switched systems, in order to further improve the performance, reliability, and safety of the critical engineering systems. It is also worth noting that complex physical systems are hard to model. The phenomenon of asynchronous switching adds further to the complexity of analytical model of such systems. Motivated by recent developments in data-driven technology for complex dynamical systems, the development of data-driven FD schemes for asynchronous switched systems becomes an attractive choice to explore in the future.

APPENDIX

A. HiMAT Model and Filter Parameters

1) Dynamics of HiMAT Switched Systems:

$$\left[\begin{array}{c|c|c|c} A_1 & B_1 & B_{d1} & B_{f1} \\ \hline C_1 & D_1 & D_{d1} & D_{f1} \end{array}\right] = \left[\begin{array}{cc|c|c|c} -1.35 & -0.98 & 1.7 & -0.1 & 1.7 \\ 17.1 & -1.85 & 0.9 & -0.09 & 0.9 \\ \hline 1 & 0 & 1.10 & 0.3 & 2.1 \\ 0 & 1 & 0.10 & 0 & 0.1 \end{array}\right]$$

$$\left[\begin{array}{c|c|c|c} A_2 & B_2 & B_{d2} & B_{f2} \\ \hline C_2 & D_2 & D_{d2} & D_{f2} \end{array}\right] = \left[\begin{array}{cc|c|c|c} -1.87 & -0.98 & 1.9 & 0.2 & 1.9 \\ 12.6 & -2.63 & 3.8 & 0.1 & 3.8 \\ \hline 1 & 0 & 1.9 & 0.4 & 1.9 \\ 0 & 1 & 2.30 & 0.9 & 2.3 \end{array}\right].$$

2) Designed Parameters of Filter for HiMAT:

$$L_1 = \begin{bmatrix} -0.8417 & 0.6766 \\ -0.4341 & 0.1159 \end{bmatrix}, \quad H_1 = \begin{bmatrix} 0.4568 & 0.4065 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} -0.4050 & -0.5047 \\ -0.8102 & -1.0042 \end{bmatrix}, \quad H_2 = \begin{bmatrix} 1.1359 & -0.5035 \end{bmatrix}.$$

B. Boost–Buck Converter Model and Filter Parameters

1) Dynamics of Boost–Buck Converter Switched Systems: According to the design, the dynamics of boost–buck converter in a matrix form are given as

$$A_1 = \begin{bmatrix} -2000 & 500 & -50 \\ 500 & -2000 & 0 \\ 50\,000 & 0 & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 50 \\ 0 \\ 0 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 1 & 0.1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$B_{d1} = \begin{bmatrix} -0.1 & 0.03 \\ -0.2 & 0.1 \\ -0.01 & 0.1 \end{bmatrix}, \quad D_{d1} = \begin{bmatrix} 0.02 & -0.1 \\ 0.01 & -0.2 \\ -0.01 & 0.02 \end{bmatrix}$$

$$B_{f1} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad D_{f1} = \begin{bmatrix} 1.5 & 1.10 \\ 1.70 & 1.90 \\ 0 & 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -1500 & 0 & 0 \\ 0 & -1500 & -50 \\ 0 & 50\,000 & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ -50 \\ 0 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0.9 & 0.5 \\ 0.4 & 0.7 \\ 0 & 0 \end{bmatrix}$$

$$B_{d2} = \begin{bmatrix} -0.01 & -0.03 \\ 0.1 & -0.16 \\ -0.2 & 0.1 \end{bmatrix}, \quad D_{d2} = \begin{bmatrix} 0.11 & 0.3 \\ 0.2 & -0.01 \\ -0.1 & -0.05 \end{bmatrix}$$

$$B_{f2} = \begin{bmatrix} 1.3 & 0.6 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad D_{f2} = \begin{bmatrix} 1.9 & 1.5 \\ 0.4 & 0.7 \\ 0 & 0 \end{bmatrix}.$$

2) Designed Parameters of Filter for Boost–Buck Converter:

$$L_1 = \begin{bmatrix} -0.2041 & -0.4082 & 0 \\ 1.7347 & -1.5306 & 0 \\ 0 & 0 & 0.0938 \end{bmatrix}$$

$$H_1 = \begin{bmatrix} 1.9388 & -1.7347 \\ -1.1224 & 1.5306 \\ -1.0575 & 0.9973 \end{bmatrix}^T$$

$$L_2 = \begin{bmatrix} -0.9178 & 1.1096 & 0 \\ 0.5479 & -2.6027 & 0 \\ 0 & 0 & 0.1143 \end{bmatrix}$$

$$H_2 = \begin{bmatrix} 0.9589 & -0.5479 \\ -2.0548 & 2.6027 \\ 3.6766 & -4.4402 \end{bmatrix}^T.$$

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Muhammad Taskeen Raza received the B.Sc. degree in electrical engineering, and the M.Sc. degree in electrical engineering from the University of Engineering and Technology, Lahore, Pakistan, in 2004 and 2011, respectively. He is currently pursuing the Ph.D. degree with the Department of Electrical Engineering, Pakistan Institute of Engineering and Applied Sciences, Islamabad, Pakistan.

He was with Cybersoft Technologies, Lahore, and Orgosoft, Lahore. He has been with the Department of Electrical Engineering, Lahore College for Women University, Lahore, since 2006, as a Lecturer. His current research interests include, fault diagnosis, fault tolerant control, autonomous systems, system modeling, and system identification.

Abdul Qayyum Khan received the B.Sc. degree in electrical engineering from the NWFP University of Engineering and Technology, Peshawar, Pakistan, in 2002, the M.Sc. degree in systems engineering from the Pakistan Institute of Engineering and Applied Sciences (PIEAS), Islamabad, Pakistan, in 2004, and the Ph.D. degree in electrical engineering from the University of Duisburg-Essen, Essen, Germany, in 2010.

He is currently an Associate Professor with the Department of Electrical Engineering, PIEAS. His current research interests include fault diagnosis in technical processes, linear and nonlinear observer design, robust control of nonlinear systems, and linear matrix inequality-based optimal design.

Ghulam Mustafa (M'06) received the B.Sc. degree in electrical engineering from the University of Engineering and Technology, Lahore, Pakistan, the M.Sc. degree in systems engineering from the Pakistan Institute of Engineering and Applied Sciences (PIEAS), Islamabad, Pakistan, and the Ph.D. degree in control systems from the University of Alberta, Edmonton, AB, Canada, in 2002, 2004, and 2012, respectively.

He is currently an Associate Professor with the Department of Electrical Engineering, PIEAS. His current research interests include sampled-data control, robust control, and application of control theory to robotics and power systems.

Muhammad Abid received the M.Sc. degree in systems engineering from the Pakistan Institute of Engineering and Applied Sciences (PIEAS), Islamabad, Pakistan, in 2004, and the Ph.D. degree from the Institute of Automatic Control and Complex Systems, University of Duisburg-Essen, Essen, Germany, in 2010.

He is currently an Assistant Professor with PIEAS. His current research interests include model-based fault detection in nonlinear systems, and robust and optimal control.