Fault Diagnosis and Fault Tolerant Control of Switched Dynamical Systems

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Thesis Submission Approval

This is to certify that the work contained in this thesis entitled **Fault Diagnosis and Fault Tolerant Control of Switched Dynamical Systems**, was carried out by **Muhammad Taskeen Raza**, and in my opinion, it is fully adequate, in scope and quality, for the degree of **Ph.D.**

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Fault Diagnosis and Fault Tolerant Control of Switched Dynamical Systems

Muhammad Taskeen Raza

Submitted in partial fulfillment of the requirements
for the degree of Ph.D.
March, 2016

Department of Electrical Engineering
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Dedications

To my mother, father, brother, sisters, wife, sons; Sarmad and Muslim;

and

Amma Ninna (May her soul rest in peace)

Acknowledgements

All praises and thanks to the almighty ALLAH SWT, Al-Rehman wa-Al-Raheim, for blessing us with knowledge and endowing the status of super creature. ALLAH SWT has blessed me throughout my life despite my limitations, and gave me the ability to undertake such a challenging task and proceed towards completion.

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Finally, I am grateful to my parents and my whole family, without their prayers, love, and support, this success could not have been possible.

(Muhammad Taskeen Raza)

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Declaration of Originality

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14 March, 2016

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Abstract

Modern technological systems consist of many components with strong interactions between them. Faults may cause an unacceptable loss of the system functionality, instability, or fatality. Systems capable of automatically detecting, diagnosing faults, and maintaining the overall functionality are desirable. Fault diagnosis (FDD) is a process that detects, locates, and finds nature of fault. Fault tolerant control (FTC) system has the ability to tolerate faults.

Among dynamical systems, switched systems (SS) have numerous applications in control of robotics, automotive industry, aircraft and air traffic control, industrial electronics (power converters) etc. A typical SS is composed of a family of subsystems and a rule that governs the switching among them. Ideally, the FDD/FTC systems are designed on the basis of assumption that it is switching synchronously with corresponding subsystems of SS; that is; FDD/FTC system switches exactly at the time of switching in the switched system to be monitored. However, in practice, the switching in FDD/FTC system lags the switching of the switched system to be monitored. This creates a particular interest in the design of FDD/FTC systems especially when there is event-based switching. In this dissertation, the term"asynchronous" is used to illustrate this situation.

This thesis studies the design of FDD and FTC systems of SS under asynchronous switching scenario, in the presence of disturbances and noise (unknown inputs). In the first part, a framework for fault detection and isolation (FDI) is proposed. The residual (symptom signal) is so generated that it is sensitive to faults and robust against disturbances. A multi-objective problem is formulated based on H_-/H_∞ filtering. Using the average dwell time approach and the piecewise Lyapunov function technique, sufficient conditions are suggested in terms of linear matrix inequalities (LMIs) to guarantee the stability and desired performance. In addition, the proposed framework has also been extended to design FDI strategy for uncertain SSs. A norm-bounded uncertainty is considered. To improve the FDD capability adaptive threshold scheme is developed.

In the second part, fault estimation (FE) and FTC schemes are proposed. The proposed framework is based on unknown input observer (UIO) and H_∞ optimization. On the basis of FE, reconfiguring control law approach is utilized to tolerate faults. To this end, an integrated approach for FE/FTC is proposed for SSs.

The last part of this dissertation addresses another very important problem of highly practical interest; that is, the design of fault detection (FD) scheme for switched system with state delays, under asynchronous switching. The tools from robust control theory, Lyapunov stability theory, and linear matrix inequality are used to propose the schemes.

To demonstrate the effectiveness of the proposed schemes, the algorithms have been tested on the dynamics of highly maneuverable aircraft technology (HiMAT) and battery converter unit (BCU) of hybrid electric vehicle.

List of Publications

Journal Publications

- Muhammad Taskeen Raza, Abdul Qayyum Khan, Ghulam Mustafa, and Muhammad Abid "Design of Fault Detection and Isolation Filter for Switched Control Systems Under Asynchronous Switching", *IEEE Transactions on Control Systems Technology*, 2015, doi: 10.1109/TCST.2015.2416314, vol. PP, Issue:99. (impact factor: 2.521)
- Muhammad Taskeen Raza, Abdul Qayyum Khan, Muhammad Abid, and Ghulam Mustafa, "Fault Detection and Isolation for Battery-Converter Unit of Drive Train in Hybrid Electric Vehicles ", *IEEE Transactions on Vehicular Technology*, under-review. (impact factor: 1.978)
- Muhammad Taskeen Raza, Abdul Qayyum Khan, Ghulam Mustafa, and Muhammad Abid, "Integrated Fault Estimation and Active Fault Tolerance in Switched Systems under Asynchronous Switching ", *Nonlinear Analysis: Hybrid Systems*, under-review. (impact factor: 2.411)

Conference Publications

- Muhammad Taskeen Raza, Abdul Qayyum Khan, Ghulam Mustafa, and Muhammad Abid "Robust Fault Detection and Isolation for Uncertain Switched Systems under Asynchronous Switching with Adaptive Threshold ", *in 9th IFAC Symposium on Fault Detection, Supervision and Safety for Technical Processes (SAFEPROCESS'15) Paris, France*, September 2-4, 2015.
- Muhammad Taskeen Raza, Abdul Qayyum Khan, Muhammad Abid, and Ghulam Mustafa, "Design of Fault Detection Filter for Switched Time Delay Systems Under Asynchronous Switching ", submitted, *55th Conference on Decision and Control (CDC), Las Vegas, USA, Dec, 2016*

- Muhammad Taskeen Raza, Moazzam Nazir, Muhammad Sabeeh, Abdul Qayyum Khan, Muhammad Abid, and Ghulam Mustafa "Fault diagnosis for reliability of wind power systems ", *in 1st Young Engineers International Convention (IYEC14)) UET Lahore, Pakistan*, April 18-20, 2014.

List of Abbreviations and Symbols

Symbol	Nomenclature
$\sigma(\mathbf{t})$	Switching signal for the subsystems
$\sigma'(\mathbf{t})$	Switching signal for the filters
$\mathbf{A^T}$	Transpose of Matrix A
$\mathbf{A}^{\dagger}$	Pseudo-inverse of a matrix A
$\mathbf{A}^{\perp}$	Orthogonal of a matrix A
ADT	Average Dwell Time
BCU	Battery Converter Unit
HEV	Hybrid Electric Vehicle
FD	Fault Detection
FI	Fault Isolation
FDI	Fault Detection and Isolation
FE	Fault Estimation
FDD	Fault Diagnosis
FTC	Fault Tolerant Control

Chapter 1

Introduction

This chapter describes the motivation, background, problem statement, objectives, and contribution of this thesis.

1.1 Motivation

Higher performance requirement and product quality are the driving forces for the development of modern systems and processes. In addition, reliability is a demand from manufacturers in order to meet the requirements of the their clients. To fulfill the specifications, engineering designs are being incorporated with sophisticated and modern components. However, in spite of all the efforts made in design and development, occurrence of fault is inevitable in every system. Consequently, it is of great importance to include fault diagnosis (FDD) system in the design of reliable systems. Fault diagnosis is a procedure to detect, locate, and find nature of fault in a system. Furthermore, fault tolerant control (FTC) design is almost obligatory in critical applications (for instance, airplanes and nuclear power plants) to ensure safety. Fault tolerant control is a technique to handle a fault in such a way that system operates similar to that in normal condition, or at least continue to perform with reduced performance, instead of quitting right away.

Switched systems are a class of dynamical systems, having more than one operating modes, for example, thermostat, and switching power converters. Switched dynamical systems have attracted much interest from the control community, not only because of their inherent complexity, but also due to the practical importance with a wide range of their applications in nature, engineering, and social sciences. On one hand, a stable switched system may become unstable while switching takes place. Remarkably, an unstable switched system may get asymptotic stability by designing a suitable switching signal [4]. On the other hand, one of the approaches to study complex and/or uncertain systems, is to transform them into equivalent switched system model. Along with these theoretical challenges and practical applications, the problem of fault diagnosis and tolerant control for switched systems is more demanding and interesting.

While studying FDD/FTC for switched systems, one of the issues is that there is difference in behaviour of each mode of the switched systems. Designing FDD/FTC for one mode may not be equally effective for the other modes. Therefore, an intuitive approach is to design FDD/FTC for each mode exclusively. In addition, the problem of asynchronous switching between FDD/FTC and modes of switched systems is undergone when the switching sequence and/or switched time of modes is unknown. Where "asynchronous" means there is difference in switching instances of FDD/FTC and modes of switched system. During unmatched period (when corresponding FDD/FTC is not intact with the mode) stability and performance issues may arise. Furthermore, occurrence of fault in this scenario puts forward challenging task of designing not only fault diagnosis but also fault tolerant control for switched systems.

The problem can be resolved by devising such a FDD/FTC system that the overall system is stable during each matched and unmatched periods, timely detects, and tolerates the occurrence of fault.

To further elaborate the research topic, background of fault diagnosis, fault tolerant control, and switched systems is discussed in the following section.

1.2 Background: Research Topic

The study of FDD/FTC is encompassed by the knowledge domains of linear algebra, physics based rules for modeling systems, analysis/design of systems, state estimation, and signal processing. One of the major areas of this study is switched systems, for which fault diagnosis and fault tolerant control design is considered.

Switched systems are a class of hybrid systems, consisting of modes/subsystems and switching signal. Modes/subsystems have either continuous-time or discrete-time dynamics. Switching signal governs the activation of any particular mode/subsystem along the trajectory of the switched system. Switching signals are mainly of the types: time-dependent, state-dependent (event-based), autonomous, and controlled. In hybrid systems, modes/subsystems of both types (continuous-time and discrete-time dynamics) coexist and interact.

Traditionally, fault detection (FD) is carried out by using redundant hardware components in a system. The idea behind the strategy is very simple and intuitive. A redundant component runs in parallel with normal operating component. Outputs from both the components are observed and any un-permitted deviation from the normal behaviour results into fault. This strategy is quite reliable, efficient, and provides fault isolation (location) in the system very easily. However, the drawbacks of additional cost, weight, and space are inherent to this strategy. To overcome the drawbacks of hardware redundancy, while achieve the attractive features of it, concept of *software redundancy* was introduced in late 70's. In this approach, instead of extra hardware component, a model is run in parallel with original component. This model is implemented at the same computer where controller of the systems is developed. In this way, there is no extra cost, weight or space incurred. This approach is also knows as (1) *model-based fault detection (FD)*, and (2) *analytical redundancy* [5]. Model-based FD approach has been an attractive choice in academia and industry for the last few decades. Due to active research developments in the field, now model-based FD approach can be further classified as: observer-based FD; parity-based FD; and parameter estimation-based FD techniques. In observer-based FD the states and/or outputs of the original system are reconstructed using state observer or output observer. Fault detection filter (a type of state observer) and diagnostic observer (a type of output observer) are widely used for the FD purpose. Observer-based FD enjoys more advantages of model-based FD when compared to other model-based FD techniques. Firstly, observer-based FD has been developed in the framework of well-established control and observer theory. Secondly, design of observer can be efficiently integrated with design of controller of the system. Thirdly, there are inherited features of quick detection, easy online implementation, and no requirement of excitation signal. In addition, it has also been shown that other model-based FD techniques are only special cases of the observer-based FD technique [5].

To meet the demands of reliability and safety of systems and personnel, fault tolerant control (FTC) has gained considerable research attention during last few decades. Fault tolerant control is the next major phase in the design of reliable and safe systems. It is designed on the basis of fault estimation (FE). Fault estimation (reconstruction) is the process of finding the nature of fault, that is, magnitude, type and, temporal features of fault. After, the fault is estimated, FTC system is designed such that it compensates (tolerates) the effects of faults. Thus, complete design chain of a fault management system comprises of fault detection, fault isolation, fault estimation and fault tolerant control. FTC systems can be categorized into two major types: passive FTC and active FTC. In passive FTC (PFTC), control law is set such that it can accommodate some known faults. For *a priori* known faults, a fixed parameter controller is set up which controls the nominal as well as faulty systems without any difficulty. This strategy is implemented using simultaneous stabilization methods, or robust control (H_∞ , disturbance rejection, etc.) based methods. These techniques are also knows as reliable control strategies. However, PFTC strategies have limited scope because all the probable faults cannot be known *a priori*. To overcome this limitation, active FTC (AFTC) is employed.

AFTC strategies can further be divided into two major types: reconfiguration, and restructuring. *Reconfiguration:* If it is possible to maintain performance of faulty systems close to that of nominal system, then controller parameters are adjusted online according to fault magnitude and type. *Reconstruction:* If more critical fault occurs, for instance, complete loss of an actuator, due to which maintaining nominal performance is not possible, then system structure or control objectives are modified to obtain reduced performance [6].

1.3 Problem Statement and Research Hypothesis

The problem of fault diagnosis and fault tolerant control for switched systems in asynchronous paradigm is studied in this thesis. In order to keep this research effort manageable, linear continuous-time switched system dynamics are considered. Average dwell time (ADT) switching constraint is assumed. At the phase of FTC design, reconfiguration strategy is utilized when there is malfunction in actuators or sensors of the switched systems. Furthermore, the effect of unknown inputs and model uncertainties on fault detection performance is also scope of exploration.

In this scenario, research hypothesis is derived as, *design of fault diagnosis and fault tolerant control systems for switched systems in asynchronous switching paradigm, while considering ADT switching, under the influence of unknown inputs and model uncertainties.*

This hypothesis will be investigated and tested through research activities: Fault detection and isolation (FDI); threshold computation; fault estimation; and fault tolerance. These activities, which are components of complete design chain of fault monitoring and management system, constitute the research process and define research objectives, which are described below:

- **Objective 1:** Design of observer-based residual generator, which in the presence of disturbances, measurement noises, and model uncertainties, successfully detects and isolates (locates) the faults in the switched systems.

- **Objective 2:** Design of suitable threshold such that contrary effects of unknown inputs and model uncertainty on fault diagnosis are reduced.

- **Objective 3:** Development of a fault estimation strategy for the problem of asynchronous switched systems. Further, on the basis of fault estimation, to propose a reconfiguration strategy for fault tolerance.

- **Objective 4:** Design a fault detection scheme for switched time-delay systems.

1.4 Contribution of the Research

The research presented in this thesis, which is the major contribution in the field of FDD and FTC for switched systems, is listed below:

1. Developed a fault detection and isolation (FDI) strategy that successfully detects and isolates actuator and sensor faults in the presence of unknown inputs and norm bounded model uncertainties. H_{-}/H_{∞} filtering approach has been developed for optimizing the effect of faults and unknown inputs on residual. Effectiveness of the derived results was demonstrated on the case studies of switching power converters and highly maneuverable technology vehicle (HiMAT).

2. Designed an adaptive threshold scheme to improve fault detection in case of uncertain switched system. Norm bounded model uncertainty was assumed in boost-converter switched system.

3. Proposed a fault estimation approach using H_{∞} filtering approach. Then, on the basis of estimated fault, an active fault tolerant control strategy was developed by "reconfiguration" of control law.

4. Proposed a strategy for fault detection in switched time-delay systems.

1.5 Overview of the Dissertation

The remainder of the dissertation is organized into the following chapters.

Chapter 2: Review: Fault Diagnosis, Fault Tolerant Control, and Switched Systems

To set a clear picture of research background, introduction to fault diagnosis, fault tolerance, and switched systems is presented in detail. Basic concepts, fundamental definitions and major issues of the research area are elaborated. Specifically, the core topics of the field; fault, failure, unknown inputs, fault detection, fault isolation, fault estimation, fault tolerance, classification of faults, classification of fault detection and tolerance approaches are discussed. In the second part of the chapter, Definition, background and fundamental concepts related to switched systems and this research are presented. Switching schemes, average dwell time, and stability problems of switched systems are discussed. Preliminaries, including the asynchronous switching problem is introduced.

Chapter 3: Fault Detection and Isolation in Switched Systems

After the foundation is laid in Chapter 2, problems of FD and FDI for switched systems are addressed in this chapter. Problems are considered under asynchronous switching paradigm. To achieve the fault detection results, H_∞ filtering approach is proposed while for FDI problem, H_-/H_∞ filtering approach is proposed. Approaches are developed in the form of linear matrix inequalities while considering average dwell time switching constraint. Obtained results are simulated on the case studies; battery converter unit (BCU) of hybrid electric vehicles (HEVs), HiMAT, and Buck-boost converters. Actuator and sensor faults are studied for the cases.

Chapter 4: Fault Estimation and Tolerance in Switched Systems

In the second phase of the thesis, fault estimation is discussed in this chapter. An approach for fault estimation is presented. To estimate faults, a design of unknown input observer (UIO) is developed. Further, fault tolerant control scheme is derived by using the "reconfiguration" strategy.

Chapter 5: Fault Detection in Switched Time-Delay Systems

After discussing results for switched systems, this chapter is devoted for a class of switched systems, switched time-delay systems (STDS). A strategy is proposed for fault detection in STDS.

Chapter 6: Conclusion and Future Recommendations

Concluding remarks and future recommendations are given in this chapter.

Chapter 2

Review: Fault Diagnosis, Fault Tolerant Control, and Switched Systems

In this chapter, framework of the thesis is established. Fault diagnosis (FDD) and fault tolerant control (FTC) areas of the research are introduced. Basic concepts, widely used model-based fault detection and fault tolerant control techniques are presented. Definitions of fault, failure, malfunction, unknown inputs Types of faults from structural and typical point of views are presented. Fault detection approaches; hardware redundancy, plausibility test, signal-based fault detection and mode-based fault detection are introduced. Pros and cones of the approaches are discussed. Observer-based fault detection, which is the focus of the thesis, is discussed in detail. Fault tolerant control is introduced with the passive FTC and active FTC types. Switched systems, which is the central component of the research process, is introduced. Reviews on stability of switched system; and FDD/FTC for switched systems, are presented.

2.1 Basic Concepts of Fault Diagnosis (FD)

A fault is a phenomenon which changes the behaviour of the system from the nominal one. A fault may cause structural or parametric changes which make the system show abnormal behaviour. The behaviour of the system is defined as the subspace containing all the input/output pair (I/O pair). If for the input $u_1(t)$, system responds with output $y_1(t)$, then (u_1, y_1) is one of all the possible I/O pairs in behaviour, B, of the system. Simply, if we take the example of a single input single output system (SISO),

$$y(t) = ku(t)$$

where k is some constant. Then the normal behaviour of the system will be a straight line, consisting of all points satisfying the system definition. In case of fault, there may exist one or more I/O pairs such that they do not satisfy the system and hence are not on the straight line, depicted in figure 2.1.

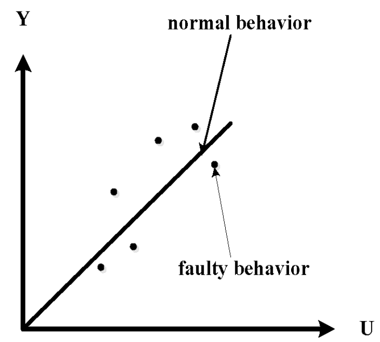

Figure 2.1: Behaviour of the system

To understand further the basic concepts related to fault, failure, disturbances, fault diagnostic, fault tolerant control etc., the well-known laboratory prototype of Three-tank system is discussed. Three-tank system [5] is the manifestation of a general process in industrial systems. The basic purpose is to maintain the level of fluid in Tank 3. Faults of different types may occur, due to many reasons, in the system. The faults of leakage may occur in any tank or connecting pipe. There may be blockage fault at any position in any pipe. Faults may also affect the pumps, the sources of fluid, or the level sensors, which measure the current level in the Tank 3.

2.1.1 Fault, Failure and Malfunction

A fault is an un-permitted deviation of at least one characteristic property (feature) of the system from the acceptable, usual standard condition [7]. Fault is a state of system that may develop abruptly (stepwise) or incipiently (drift wise). The nature of fault depends on the cause of fault. Fault may occur due to shortcomings at design level, at manufacturing time, or assembling time. At field, fault occurrence may be caused due to wrong operation (overload) or normal operation (wear). One of the reasons of fault is human error while working at system. Un-permitted deviation is the difference between faulty value and the tolerance value of normal operating zone. Features of the system which are affected by fault are usually proportional to magnitude of fault. Faults of smaller size may not be detected properly. Faulty state in the system may result in two cases; either failure or malfunction. A failure is a permanent interruption of a system's ability to perform a required function under specified operating conditions. A malfunction is an intermittent irregularity in the fulfillment of a system's desired function. Failure or malfunction are the events, which are result of faulty state in the system. These two events usually occur after the start of operation, when there is continuous increase in the stressing of system.

2.1.2 Types of Faults

Again referring back to Three-tank system, physically faults can be categorized into actuator, component, and sensor faults.

Actuator Faults: The faults, which appear in the actuator of the systems are actuator faults. In Three-tank system, fault in the pumps and related mechanism are actuator fault. In general, examples of actuator faults are; in battery of boost-converter, in control rod of nuclear reactor, and in motor of elevator.

Components Faults: The faults, which occur in the actual systems structure are called component faults. In Three-tank system, faults in tanks and pipes are component faults. In general; faults in resistor, capacitor or inductor of boost-converter, in reactor core of nuclear reactor, in springs of mass-damper system are examples of component faults.

Sensor Faults: The faults, which affect the sensors/measuring instruments of the system are called sensor faults. Fault in the level sensor of the Three-tank systems is sensor fault. Other example of sensor faults are; voltage sensor of boost-converter, core temperature sensor of nuclear reactor.

Characteristically, faults can be classified into abrupt, incipient or intermittent faults.

Abrupt Faults: This type of fault appears suddenly in the system. Abrupt faults are usually severe, however these can be detected easily.

Incipient Faults: This type of fault grows gradually in the systems and is relatively hard to detect.

Intermittent faults: Intermittent faults manifest themselves in intervals. These faults are also easier to detect. For further details, please refer to [8].

2.1.3 Unknown Inputs

It is important to differentiate between fault and unknown inputs. Unknown inputs are unwanted or not desired sort of things which are not faults but should be avoided for proper operation of the system. Further, if these unwanted things are of severe nature then it may result in fault or failure of the system. Environmental disturbances, unexpected changes within process variables during operation and process/measurement noise are commonly termed as unknown inputs. Example of disturbance: static and coulomb frictions in inverted pendulum are the factors which may change the system dynamics unpleasantly.

2.1.4 Fault Monitoring and Management Systems

Depending on application and severity of faults, different levels of solution can be employed in fault monitoring and management systems. Depth of implementation of the different levels may possess functions; fault detection, fault isolation, fault estimation, and fault tolerance. In the sequel, we discuss these functions.

Fault Detection (FD)

Fault detection is the process to detect the occurrence of fault. Output of fault detection system is ON/OFF alarm signal.

Fault Isolation (FI)

Fault isolation is the process of finding the location of the fault. Output of fault isolation system is ON alarm signal for faulty part/component.

Fault Detection and Isolation (FDI)

Fault detection and Isolation (FDI) is the process, consisting of fault detection (FD), and fault isolation (FI) capabilities.

Fault Estimation (FE)

Fault estimation is the process of determining the nature (magnitude with temporal behaviour) of the fault. Output of fault estimation system is estimated fault signal. Fault estimation is also called fault identification.

Fault Diagnosis (FDD)

Fault diagnosis (FDD) is the process, consisting of fault detection (FD), fault isolation (FI), and fault estimation (FE) capabilities.

Fault Tolerance (FT)

Fault tolerance is the process of tolerating (compensating) the faults in the system. Output of the fault tolerance system is normal operating behaviour of the compensated faulty system.

2.1.5 Fault Diagnosis Approaches

There are many fault diagnosis techniques being developed and employed in academia and industry for reliable and safe processes. In this section, these approaches are presented as follows.

Hardware Redundancy

The main theme of this technique is to use redundant hardware component for the part of the system for which reliability is desired. Fault detection logic is basic and intuitive. Whenever there is discrepancy between original component and its redundant component, fault is detected by observing difference between them. Fig. 2.2 shows the theme pictorially. Major advantages owned by the techniques are; it is highly reliable and fault is located (fault isolation) in straightforward way. However, these advantages are gained with the heavy price in form of extra hardware cost, extra maintenance and operation cost, and extra space requirement. Due to these pros and cones the technique is suitable for limited applications. The applications for which safety is much important than cost, for example, nuclear power plants, aircrafts, and air traffic control systems [5, 9].

Plausibility Test

The basic idea behind the approach of plausibility test is on the notion that system components operate according to some physical laws, convincing values and/or their consistent mutual interaction. According to this notion, whenever there is inconsistency between the measured process variables and plausibility rules, it is indication of faulty situation. Fig. 2.3 shows the idea of the technique.

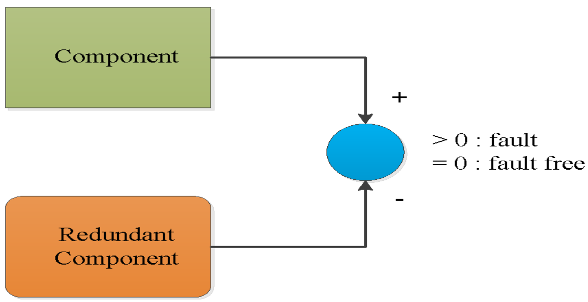

Figure 2.2: Hardware redundancy based fault detection

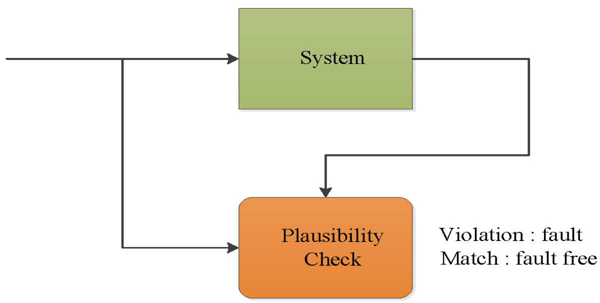

Figure 2.3: Plausibility test based fault detection [1]

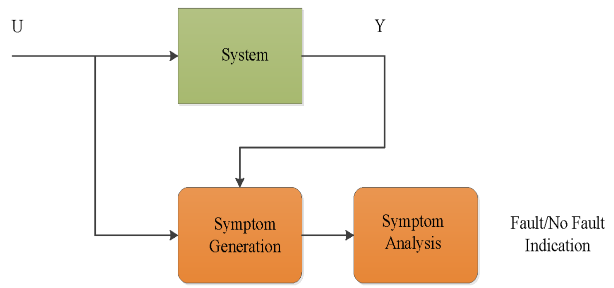

Figure 2.4: Signal-based fault detection

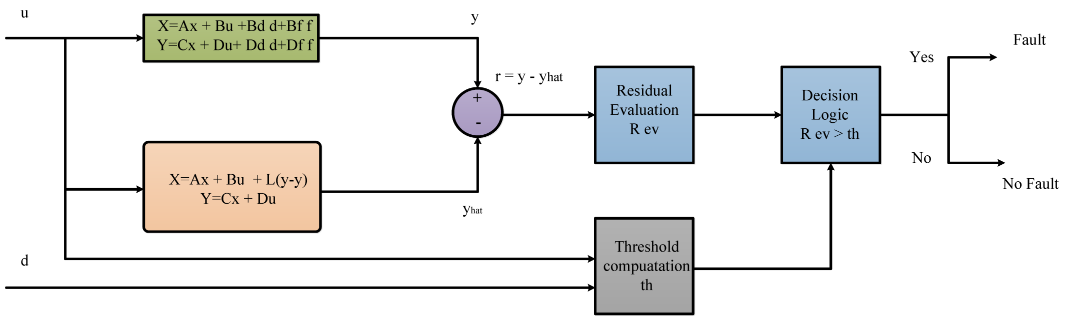

Figure 2.5: Model-based residual generation and evaluation

Plausibility test technique can be viewed as basic step towards development of model-based fault detection approach. However, the scope of this test is limited to simple systems [5, 7]. Benefit of the technique are; simple and low in cost.

Signal-based Fault Detection

The central concept of the signal-based FD is linked with the theory that there may be some measurable signals which carry the fault information. Whenever, there is fault in any part of the process, the related signals are influenced and thus carry symptoms of that fault. The symptoms can be analyzed further by using well-established signal processing techniques to know information about fault. These symptoms may be studied in time domain and/or frequency domain. Time domain symptoms are magnitude, limits and trends, means and moments, while frequency domain symptoms are spectrum, spectral power densities etc. Fig. 2.4 depicts the concept. This technique is efficient in fault detection for small operating ranges. Signal processing based FD can be employed in steady state efficiently.

Model-based Fault Detection

The in-built idea for model-based FD is inferred from the above mentioned FD approaches. By using the theme of hardware redundancy approach, model of a process or considered component is developed in software at computer. By utilizing system modeling techniques the model is established, which quantitatively or qualitatively describes the process steady state and dynamic behaviour. Thus we are able to reconstruct process or component on-line in software, and hence term the model as software redundant model or analytical redundant model. The whole idea is called *software redundancy* or *analytical redundancy*.

Similar to original hardware redundancy implementation, the process model is driven with same inputs of the process and runs side by side with original process. In this scenario, it can be reasonably projected that in fault free process, the reconstructed process variables by the process model follow the really measured process variables. In the faulty situation of process, the reconstructed variables by the process model deviate from the actually measured process variables. Rationally, to observe the deviation, it is required to compare measured process variables and reconstructed (estimated) process variables. This implementation of comparison stage is similar to plausibility test approach. The comparison between the two signal values generates a symptom signal which is called *residual*. Residual signal carries the useful information about successful fault detection. After comparison, the next intuitive phase for fault detection is to check the value of residual. Simply: if residual $= 0$, then no fault; if residual $\neq 0$, then, a fault. The above stated process of estimating the process variables and then comparing the estimates with measured variables is called *residual generation*. Correspondingly, in the framework of model-based FD, the process model and comparison unit constitute the *residual generator*.

In practice, the residual signal is not only influenced by fault but also by unknown inputs (disturbances, noises) acting from inside or from external environment of the process. In addition, there is a problem associated with model-based FD. Exact model, whether quantitative or qualitative, is hard to reconstruct. There are uncertainties involved in modeling techniques. Furthermore, in the design of fault isolation and fault estimation systems, the residual signal needs the more processing to be carried out. To resolve above mentioned issues; one approach is to post-process the residual signal. This next stage of residual processing, to get desired information about fault and its characteristics, is called *residual evaluation*. It is fair to give residual evaluation stage a resemblance with signal-based FD.

An important step at this stage is to compare residual with a tolerance level of fault free case. The maximum value of tolerating disturbances and noises in fault free case is called *threshold*. *Statistical methods* and *norm based evaluation* are the most popular approaches to achieve post processing of residual, at residual evaluation stage. Another approach to resolve issues related to model-based FD framework is to design a residual generator such that fault of interest is decoupled from other faults, unknown inputs, and model uncertainties. This approach is being actively followed by many researchers of the area. One of the schemes of this research area is observer-based FD technique.

In this research, both of the discussed approaches are explored for problem of fault diagnosis and fault tolerant control in asynchronous paradigm. The complete idea of model-based FD is shown in Fig. 2.5.

2.1.6 Model-based Fault Diagnosis

As discussed in preceding section, in model-based FD frame work, process model can be of quantitative (*analytical*) type or of qualitative (*knowledge-based*) type. In this way, model-based FD can be categorized into two major types; analytical model-based FD and knowledge-based FD. Qualitative process models are expressed in terms of qualitative functions, for examples, expert systems, petri nets, neural networks, fuzzy rules etc. Advantages of qualitative models based FD are revealed, when the process model is too complex to model, mathematical model is difficult to obtain, or the system is poor in information. A detailed survey based on this type of model-based FD can be found in papers [10] [11, 12, 13, 14, 15]. In last few years there are hybrid model-based FD approaches, based on both qualitative and quantitative models, have been proposed [16]. Analytical models, are expressed in the form of mathematical equations (differential equations). Further developments in analytical model-based FD give rise to i) observer-based approach, ii)parity space approach, and iii) parameter estimationbased approach. Observer-based approach is discussed below, while it is referred to [5] for other types of analytical model-based FD approaches.

Observer-based Approach

Observer-based approach is one of most commonly used model-based FD approach. Original idea of model-based FD; comparison between measured process variables and estimated process variables is utilized here. It is worth noticing that observers are being used in control community as well as in fault detection community. In control community, observers which are used are state observers. State observers are used to estimate the un-measurable states of the system. In fault detection community, output observers are used, to estimate output variables of the process. There is special type of observer, fault detection filter (FDF) which is common in use. FDF estimates all the states whether they are measurable or un-measurable.

Historically, idea of using observer for fault detection purpose goes back to early 70s. At that time, Beard proposed the fault detection filter, later on, the more developments were made by Jones in 1973. This filter was named *Beard-Jones fault detection filter*. Since then, observer-based FD has been the focus of FD research community. To propose solution for robustness of residual signal, against disturbances and noises, first robust FDI design for instrument fault was proposed by Frank [17]. Later, in 80s, robust unknown input observer (UIO), a pioneer work, was presented by Wunnenberg and Frank [18], followed by other contributions from researchers in [19, 20, 21, 22, 23]. The idea of UIO was presented, such that, residual signal may be generated independent of unknown inputs. The idea was successful but conservative in the sense that, existence condition was hard for such a type of proposed UIO. After that, matrix pencil approach was presented [24, 25, 26]; the idea of this approach was to make unknown inputs insensitive to residual signal instead of decoupling desired states estimated states from unknown inputs.

Another, well-designed solution to this problem was presented by Massomnia [27], using geometric approach. This technique was initially complex and not very much suitable for implementing in FD. Later the modified version of this technique has been introduced in [5]. Furthermore, different techniques for full order, reduced order and minimum order detection filters have been proposed [5].

Observer-based fault diagnosis and fault tolerance for switched system is the focus of this thesis. A brief survey on residual generation techniques for switched systems will be presented in Section 2.6, while detailed review is presented in introduction sections of the following chapters.

2.2 Basic Concepts of Fault Tolerant Control (FTC)

All technological systems, whether, simple/complex or traditional/modern are prone to faults. Fault and malfunctions can jeopardize the entire plant and personnel at work. During the last three decades, fault detection and diagnosis (FDD), that is, combination of fault detection and Isolation (FDI) and fault estimation (FE), has been addressed intensely [28, 29, 30, 5, 31, 32, 33], while fault tolerance is relatively young, nevertheless active area of the research [34, 35, 36, 6, 37, 38, 39, 40, 41]. Although, every type of deviation from normal behaviour in the system is not ignorable from the aspects of safety and reliability. Particularly, actuator faults may degrade overall system performance very quickly, as their effect is propagated throughout the system. Likewise, erroneous sensor reading may change the feedback control law unnecessarily and result in poor performance or instability [35].

Rational approach to the issue; firstly, to detect and locate as quickly as possible the faults, firstly. Secondly, if possible then, to compensate faults by any suitable means, and if not possible, to stop/shutdown the process until the maintenance of the faulty part. In the research field two methodologies are being followed. In first strategy, FTC is designed on the basis of FDD [42, 43]. In second approach, FE is performed directly and then suitable control law is designed in FTC scope [44, 45, 46]. It is discussed that second way is less conservative, direct and have greater potential in application. It is worth noticing that if it is required, FE procedure can be directed towards FD and FDI also.

In any of the two cases, FTC can be designed mainly in two ways; passive FTC and active FTC.

Passive Fault Tolerant Control

Here, "Passive" means, the controller is fixed and designed at manufacturing time while keeping in mind the some possible fault occurrence scenarios. Definitely, this approach is conservative as all the faults cannot be known a priori or it is not possible to accommodate all the faults with fixed controllers [6]. However, the approach is smooth as it does not have to bother for the issues like switching transients.

Active Fault Tolerant Control

On the other side, the term "active" in active FTC implies that controller can react and adapt itself to faulty situation in real time. In this way scope of active FTC is larger than passive FTC. However, FDI and/or FE are prerequisites for this and performance of FTC is directly related with quality of FE. Further, the FTC approach has to face the problem of switching transients in controller reconfiguration. It is important to note that FDI and/or FE are not required at all in passive FTC.

In the next sections, basic and fundamental concepts about switched systems are presented. In addition. preliminaries, including asynchronous switching problem, are discussed.

2.3 Introduction: Switched Systems

Switched systems is a class of hybrid systems. Hybrid systems are dynamical systems having continuous dynamics, and discrete events and interaction between these dynamics. Many systems and natural phenomenon can be modelled as hybrid system. An example of hybrid system is car transmission system having manual as well as automatic transmission of gears [47]. Operation principle of the vehicle is that: gears can be switched manually (discrete events) to achieve the desired velocity (continuous dynamics). It is also provision that, on the basis of velocity (continuous dynamics) gears can be changed (discrete events) automatically by the automatic transmission system. In this way, there is interaction between the continuous dynamics and discrete events.

Studies on hybrids systems lie in multidisciplinary area of control theory , computer science, mathematics. Computer scientists pay attention to the discrete events and their analysis in depth. Attraction of control community is in the continuous dynamics of the hybrid systems while assuming switching from patterns in defined set of discrete events. In other words, control community is much interested in effect of discrete events on continuous dynamics. It is also worth noticing that the scope of attention of computer scientists also constitutes a switched system; continuous dynamics changing the discrete states. That is why the switched system definition in both communities is generic. Now it can be stated clearly that motivation of this study is in the context of switched systems from the control system community point of view. For better comprehension, basic example of SS can be of modified form of car transmission system discussed as hybrid system in above. If we assume only the manual transmission system in the car, that is only gears (discrete events) can change the modes of velocity (continuous dynamics), then it is an example of switched systems.

2.3.1 Switched Systems: Definition

Dynamical systems that are described by interaction between continuous and discrete dynamics are called hybrid systems. Switched systems are a class of hybrid systems, which involves either continuous time dynamics or discrete time dynamics. Mathematically, a switched system can be described by a differential equation of the form

$$\dot{x} = f_\sigma x \tag{2.1}$$

where, $\{f_\sigma : \sigma \in \rho\}$ is a family of sufficiently regular function from R^N to R^N, that is parameterized by some index ρ; and $\sigma : [0, \infty) \longrightarrow \rho$ is a piecewise constant function of time, called a switching signal. Such a function σ has a finite number of switching times and takes a constant value on every interval between two consecutive switching times. The role of $\sigma(t)$ is to specify, at each instant t, the index $\sigma(t) \in \rho$ of the active subsystem.

2.4 Switching Types

There are different types of switched systems depending upon the switching signal, which is used for switching from one mode to another mode [48]. Main types of switching are; time dependent switching, state dependent (event-based) switching, autonomous switching, and controlled switching.

2.4.1 Time-based Switching

Consider a family of systems defined by the following model

$$\dot{x} = f_i x \tag{2.2}$$

where $i \in \{1, 2, ...N\}$.
Depending upon the index i we get different systems. N, represents the total number of modes in the system and switching from one mode to another takes place depending upon a switching signal $\sigma(t)$, which is a function of time.

$$i = \sigma(t) : [(0, \infty) \longleftrightarrow \{1, 2, ...N\} \tag{2.3}$$

Depending upon the switching signal $\sigma(t)$, switching will take place from one mode to another, and i will take values from the set $i \in \{1, 2, ...N\}$. Therefore, switched systems with time dependent switching signal can be represented by

$$\dot{x} = f_{\sigma(t)}(x(t)) \tag{2.4}$$

When all the sub-systems are linear, then we get a switched linear system of the form,

$$\dot{x} = A_{\sigma(t)}(x). \tag{2.5}$$

For further details, the reader is referred to [4],[10],[11].

2.4.2 State-based Switching

If the switching from one mode to the other mode takes place depending upon the states, then we call it state dependent switching [48, 47, 49]. As in the example of thermostat given earlier, the thermostat is turned ON/OFF depending upon the temperature, which is a state of the system. In general it can be stated that the entire state space of the switched system is divided into some operating regions by the switching surfaces. The number of operating regions of the system may either be finite or infinite [47].

2.4.3 Autonomous Switching

By autonomous switching means, when there is no direct control over the switching mechanism that triggers the discrete events. This category includes systems with state-dependent switching in which locations of the switching surfaces are predetermined, as well as, systems with time-dependent switching in which the rule that defines the switching signal is unknown (or it was ignored at the modeling stage). For example, abrupt changes in system dynamics may be caused by unpredictable environmental factors or component failures.

2.4.4 Controlled Switching

In contrast with the above, in many situations the switching is actually imposed by the designer in order to achieve a desired behaviour of the system. In this case, we have direct control over the switching mechanism (which can be state-dependent or time-dependent), and may adjust it as the system evolves. For various reasons, it may be natural to apply discrete control actions, which leads to systems with controlled switching.

2.5 Review: Stability of Switched Systems

One of the fundamental problems in studying the switched systems is the stability. This issue has been addressed in details, for instance [50, 51, 52, 53, 53, 54, 55, 56, 57, 58]. Stability of switched systems not only depends on the stability of the sub-models, but also on the switching signal, [59]. Liberzon and Morse have investigated the stability problem for switched systems in [4]. In this pioneering work, they have classified the stability problems in switched systems as follows:

- Find conditions that guarantee the asymptotic stability in switched systems for any switching signal. In other words, this problem defines the stability conditions for arbitrary switching signal, since the stability of all sub-models is not sufficient to ensure the stability of the overall switched systems.

- Identify a class of switching signals for which, a stable switched systems retains it stability. In this problem, it is assumed that all the individual sub-models are asymptotically stable.

- Design a switching sequence that ensures the asymptotic stability of switched systems. This problem is known as stabilization of switched systems.

The stability problem of the switched systems is generally studied within the Lyapunov stability framework, such as, common quadratic Lyapunov function (CQLF), switched Lyapunov functions (SLF) and multiple Lyapunov functions (MLF). More details can be found in [47, 60]. In the following two subsections, the stability will be highlighted under the arbitrary and constrained switching signals.

2.5.1 Stability Under Arbitrary Switching

When studying the stability analysis problem for switched systems, the first question which is considered is whether the switched system is stable when there is no restriction on switching signal . This problem is usually called stability analysis under arbitrary switching. Prerequisite for this problem is that all the subsystems of switched systems should be asymptotically stable. It is still possible that there is divergent trajectory for such switched systems. Therefore, in general, the above mentioned prerequisite is not sufficient to ensure stability under arbitrary switching for switched systems. There are few special cases when this prerequisite may be sufficient for overall stability of switched systems such as being pairwise commutative [61, 62], symmetric [63], or normal [64]. The existence of common Lyapunov function for all subsystems of switched systems makes sure the stability of the system under arbitrary switching. In this way, the problem of stability under arbitrary switching can be solved. Lot of efforts have been focused on the study of common quadratic Lyapunov functions for switched systems.

Common Quadratic Lyapunov Functions

Quadratic stability is a class of exponential stability, which implies asymptotic stability. Lot of research has been made on the class of stability for its importance in practical problems of stability. Existence conditions for common quadratic Lyapunov function (CQLF) can be expressed in terms of linear matrix inequalities (LMIs). In the light of above mentioned comments, it is said that, if there exist a common quadratic Lyapunov function (CQLF) for all the subsystems of switched systems, then quadratic stability of the switched system is ensured and hence exponential stability is guaranteed.

Switched Quadratic Lyapunov Functions

It is possible, in few cases, that switched system is exponential stable, without having common quadratic Lyapunov function (CQLF) [47]. This implies the conservatism of common quadratic Lyapunov function (CQLF). Due to this reason, research focus has been paid to less conservative class of Lyapunov functions, called switched quadratic Lyapunov functions [51, 52].

2.5.2 Stability Under Constrained Switching

Few of the switched systems may not be stable under arbitrary switching signals, but may be stable under constrained (restricted) switching, for instance, a closed-loop multiple controller system [60]. It is also possible that restricted switching arises naturally from the system behaviour itself, for instance, automobile gear switching. In automobile gear switching particular, switching sequence has to be followed. Moreover, in some cases it is possible to have *a priori* knowledge about switching sequence in switched systems. For instance, there may requirement of certain time bound between two successive switching modes. This *a priori* knowledge about switching may help in devising stronger stability conditions for given switched systems than in arbitrary switching signals. These above mentioned cases constitute the area of stability under constrained switching for switched systems. By solving this problem of constrained switching, it is possible to answer the question of what kind of restrictions are required to make sure the stability of switched systems. The restrictions on switching signals are mainly of two types; state space domain restrictions and time domain restrictions. Case of state space restriction; there are abstractions from partitions of the state space. Average dwell time (ADT) switching signal is an example of time domain restriction on switching signal.

Dwell Time Switching Constraint

Generation of divergent trajectories when switching between two stable subsystems takes place is caused by failure to absorb energy. This phenomenon is evident by studying example in [59, 4]. Further, in case of unstable modes, when there is an unstable subsystem (for instance, controller is failed or sensor got fault), if system remains active in unstable mode too long or it switches at times to unstable mode, then switched system may get unstable. In this scenario, a rational question arises; what would happen if switching signal is restricted to a subclasses. Intuitively, if system stays for long time in stable mode or it switches less frequently, that is, slow switching takes place, then system may get stability. These rational ideas have been proven to be quite effective in the form of dwell time and average dwell time switching concepts [54, 65, 66].

Definition 1. *(Average Dwell Time (ADT)[60]; For any switching signal* $\sigma(t)$ *and any* $t_2 > t_1 > 0$*, let* $N_\sigma(\tau, t)$ *denotes the number of switchings of* $\sigma(t)$ *in an interval* (t_1, t_2)*. If*

$$N_\sigma(t_2, t_1) \leq N_o + \frac{t_2 - t_1}{\tau_\alpha} \tag{2.6}$$

holds for a given $N_o \geq 0$ *and* $\tau_\alpha > 0$*, then the constant* τ_α *is called the average dwell time and* N_o *the chattering bound.*

The reason for a switching signal that satisfies 2.6 is considered having an average dwell time no less than τ_α is because

$$N_\sigma(t_2, t_1) \leq N_o + \frac{t_2 - t_1}{\tau_\alpha} \Leftrightarrow \frac{t_2 - t_1}{N_\sigma(t_2, t_1) - N_0} \geq \tau_\alpha$$

which means that on average the "dwell time" between any two consecutive switchings is no smaller than τ. It has been shown in [65] that if all the subsystems are exponentially stable then the switched system remains exponentially stable provided that the average dwell time is sufficiently large.

Linear parameter-varying (LPV) Systems are also one of the classes of dynamical systems, which are usually misunderstood as a class of switched system. Linear parameter-varying systems are linear time-varying systems whose state-space matrices are fixed functions of some vector of varying parameters, $\theta(t)$ [67]. Hence LPV systems are described by state-space equations of the form

$$\begin{aligned} \dot{x}(t) &= A(\theta(t))x + B(\theta(t))u \\ y(t) &= C(\theta(t))x + D(\theta(t))u \end{aligned} \tag{2.7}$$

One example of a physical system whose (linearized) dynamics take the form of a parameter-varying system is an aircraft, where the time-varying parameter is typically dynamic pressure [68]. Major difference between switched systems and linear parameter varying systems is that; in switched systems modes are changed through a switching signal, while in linear parameter-varying systems there is not any realizable switching signal present.

2.6 Review: FDD and FTC in Switched Systems

This sections gives a brief survey on FD and FTC research which has been carried out for switched systems. The survey on switched systems, in general, reveals that there is extensive research for stability and performance of switched systems. However, there are relatively few results for FD and FTC in switched systems. The detailed review is presented, at the start of the following chapters, relevant to the problems discussed in those chapters. Few results on FD and FTC for switched systems are presented next.

- The problem of FD for discrete-time switched time-delay systems has been addressed in [69]. The case is studied under arbitrary switching constraint while state delay in switched systems. In the work, FD problem has been transformed into H_∞ problem by augmenting states of system and fault detection filter. Then, conditions are formulated in the form of linear matrix inequalities by employing switched Lyapunov function. The proposed results were extended to continuous time-switched systems in [70].

- In [71] solution has been proposed for fault estimation in time delay switched systems. Problem of fault estimation was casted in linear matrix inequalities, while using switched Lyapunov function. The fault is estimated using adaptive observer.

- Work on FD for switched systems under the effect of unknown inputs while presence of uncertainties in the model, was presented in [72]. Problem was formulated by using model matching approach. Filter was designed by solving linear matrix inequalities. This work was extended for the case of unknown switching sequence (asynchronous switching) in [73].

- For the case study of turntable system, fault detection filter was proposed in [74]. The system was studied in linear continuous time domain and problem was transformed to H_∞ framework. Linear matrix inequities were derived for the design of filter. Problem was discussed in average dwell time constraint.

- In [75], a complex nonlinear system, lateral vehicle dynamics, was modeled into switched system. Then, a scheme was proposed to detect faults in the modeled switched system.

2.7 Preliminaries and Problem Formulation

2.7.1 System Description: Switched System and Switched Fault Detection Filters

In general, following class of continuous-time switched systems is considered,

$$\begin{aligned}\dot{x}(t) &= A_{\sigma(t)}x\,(t) + B_{\sigma(t)}u\,(t) + B_{d\sigma(t)}d\,(t) + B_{f\sigma(t)}f\,(t)\\ y(t) &= C_{\sigma(t)}x(t) + D_{\sigma(t)}u(t) + D_{d\sigma(t)}d\,(t) + D_{f\sigma(t)}f\,(t)\end{aligned}\tag{2.8}$$

where, $x(t) \in R^n$ is the state vector, $u(t) \in R^r$ is the control input vector, $y(t) \in R^m$ is the output vector, $d(t) \in R^p$ is the unknown inputs (disturbances, noise) vector, $f(t) \in R^q$ is the fault vector. While, $\sigma(t)$ is a switching signal which is piecewise constant function $\sigma : [0, \infty) \longrightarrow \rho$. Such a function σ has a finite number of switching times and takes a constant value on every interval between two consecutive switching times. The role of $\sigma(t)$ is to specify, at each instant t, the index $\sigma(t) \in \rho$ of the active subsystem. Also, $A_{\sigma(t)}, B_{\sigma(t)}, C_{\sigma(t)}, D_{\sigma(t)}, B_{d\sigma(t)}, D_{d\sigma(t)}, B_{f\sigma(t)}, D_{f\sigma(t)}$ are the systems, disturbances and fault coupling matrices with appropriate dimensions. We denote the association of these matrices with particular switching signal instant $\sigma(t) = i$ by $A_{\sigma(t)} = A_i$, where $i = 1, 2, ...N$, number of subsystems involved.

In order to generate residual signal, the following switched fault detection filter is used,

$$\begin{aligned}\dot{\hat{x}}(t) &= A_{\sigma'(t)}\hat{x}(t) + B_{\sigma'(t)}u(t) - L_{\sigma'(t)}(y(t) - \hat{y}(t))\\ \hat{y}(t) &= C_{\sigma'(t)}\hat{x}(t) + D_{\sigma'(t)}u(t)\\ r(t) &= H_{\sigma'(t)}(y(t) - \hat{y}(t))\end{aligned}\tag{2.9}$$

Where, $L_{\sigma'(t)} \in R^{n\times m}$ and $H_{\sigma'(t)} \in R^{q\times m}$ are the parameters of the filter to be designed with respect to each subsystem $i \in \{1, 2, ...N\}$. Similar to the system, the switching between different modes of the filter depends on the switching signal $\sigma'(t)$, shown in Fig. 2.6.

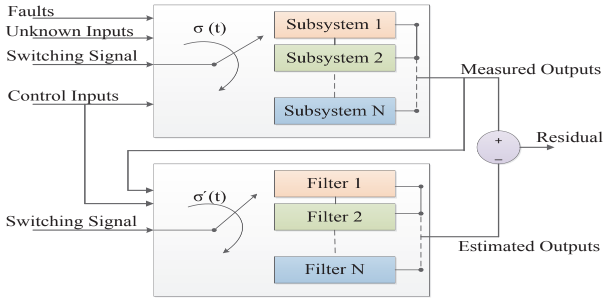

Figure 2.6: Switched system and switching fault detection filters

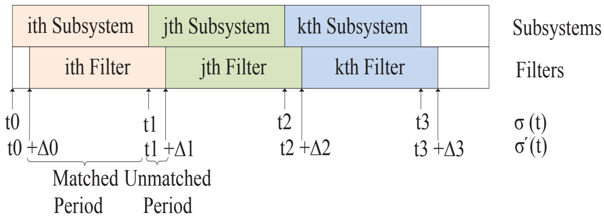

Figure 2.7: Asynchronous switching between filters and subsystems

2.7.2 Asynchronous Switching

Practically, there exists the phenomenon of asynchronous switching between the filters and the subsystem in most of the cases. Here, in this thesis, we assume the switching sequence is unknown *a priori*; in addition, it is also assumed that a module (device) is present which triggers the information about activation (switching) of subsystem. After the identification of subsystem, the corresponding filter is switched with some delay, Δ_i. Fig. 2.7 shows the phenomenon of asynchronous switching. It is easy to see that each filter lags by some time Δ_i to its corresponding subsystem. In this way, there is matched period, when ith (jth) subsystem and ith (jth) filters are in operation. On the other hand, during unmatched period jth system but ith filters are in operation, and vice versa. These periods may cause stability problem for overall system, when switching from matched period to unmatched period.

2.7.3 Assumptions

In this study, following assumptions are made, while proposing schemes for the problems of FD and FTC:

A.1 ADT switching constraint

A.2 The pair (A_i, C_i) is detectable

A.3 For (2.8), fault detectability condition,
$C_i(sI - A_i)^{-1}B_{fij} + D_{fij} \neq 0$ holds, where B_{fij} and D_{fij} denote the jth columns of B_{fi} and D_{fi}, respectively.
In addition to above mentioned assumptions, following assumptions are specific to the problems.
For fault detection and isolation (FDI) problem, considered in Chapter 3,

A.4 matrix, D_{fi} is invertible.
For fault estimation and fault tolerant problem, considered in Chapter 4,

A.5 The pair (A_i, B_i) is controllable

In general;
A.1 implies that switching speed of the system should not be faster than τ_α (ADT) when changing modes,
A.2 ensures that states of the systems can be estimated through observer,
A.3 means transfer function from fault to system output is not zero. In other words, a fault ξ_i is detectable if for some input u, $\frac{\partial y}{\partial \xi_i}\big|_{\xi_i=0} d\xi_i \neq 0$ [5].
A.4 implies that number of outputs are greater than number of faults,
and A.5 ensures that states of the systems can be controlled through a controller.

2.8 Summary

This chapter seeks to discuss background knowledge and literature survey of fault diagnosis, fault tolerant control, and switched systems. Basic concepts, major techniques and approaches related to FDD and FTC of switched systems are discussed. Then, background knowledge and fundamentals of switched systems have been presented. Definition and examples of switched systems, types of switching signals, reviews on switched system stability, and FDD/FTC for switched systems are presented. At the end of the chapter, preliminaries and problem of asynchronous switching is introduced to complete the state of the art for the research of the thesis.

Chapter 3

Fault Detection and Isolation in Switched Systems

In this chapter, fault detection and isolation for switched systems under asynchronous switching is discussed. Different schemes, based on fault detection filters, are proposed to detect and isolate the faults.

3.1 Introduction

The problems of fault detection and isolation, in the presence of disturbances and noise, for continuous-time linear switched control systems are addressed in this chapter. The residual generators are proposed, which are based on asynchronously switching filters. Problem of fault detection in switched systems is formulated into H_∞ filtering problem. In proposed H_∞ technique, residual is generated, such that, it is robust against process disturbances and measurement noise. To address the fault detection and isolation issue, the problem is formulated as mixed H_-/H_∞ filtering problem. In proposed H_-/H_∞ technique, residual is generated such that, it is sensitive to faults and robust against process disturbances and measurement noise. In addition, proposed filter has prominence of granting fault isolation capability along with fault detection. Next, the FDI problem is discussed for uncertain switched systems, while considering norm bounded uncertainty. In the above mentioned discussions, to deal with the issue of asynchronous switching, during matched and unmatched time of switched systems, a piecewise Lyapunov function along with average dwell time scheme is employed, and sufficient conditions are derived in terms of linear matrix inequalities. To simplify the application procedure, algorithms are also presented in the light of proposed frameworks. Then, proposed schemes are designed for case studies of, battery converter unit of hybrid electric vehicle, Highly Maneuverable Aircraft Technology (HiMAT), and Buck-boost power converter. Finally designed filter parameters are simulated to illustrate the efficacy of the proposed frameworks.

Fault detection and isolation (FDI) for dynamical systems and processes has been of considerable interest during the last three decades. For this purpose a great deal of attention has been paid to fault detection and isolation schemes [76, 5, 77, 9, 78, 79, 80, 81, 82, 83, 84, 85, 86]. Among various fault detection (FD) approaches, model-based FD and data-driven FD are receiving considerable attention in the recent research. In data-driven framework [78, 82, 87] the FD system design relies on input-output data only to generate symptom signal, which carries the fault information. In case of the availability of system model, the model-based FD [5, 9, 83] is an attractive choice for efficient detection of faults. One of the problems in this research domain is the coupling of desired faults with unknown inputs (disturbances, noise, and uncertainties), which are the major source of false alarms in industrial systems. To cope with this problem, the FDI system has to be maximally sensitive to faults occurring in the system and at the same time maximally robust against unknown inputs. To this end, a range of optimization indices have been proposed. Few of those are H_2/H_2 [5], H_∞/H_∞ [9], H_-/H_∞ [86, 88], H_-/H_∞ index is of particular interest in this research, because of its twofold nature for the solution of the said problem. The H_- index takes into account the minimum influence of faults, whereas the H_∞ norm considers the worst-case effect of unknown inputs on the residual signal.

On the other side, due to their significance in theory and practical applications switched systems have fascinated many researchers. These systems have numerous applications in control of robotics, mechanical systems, automotive industry, aircraft and air traffic control, switched power converters, and in many other fields [47, 89, 70, 90, 73, 71, 91, 74, 92, 93]. It is also worth noting that stability problems of switched systems are unique and also complex. Individually stable subsystems may become unstable at system level when switching takes place among them.

On the other hand, unstable subsystems may be stabilized by choosing a suitable switching scheme. This critical situation turns into rather severe nature, when a fault occurs in any component of a switched system.

Currently, it is an active research area to explore the problem of fault diagnosis for switched systems. Fault detection (FD) problem for discrete time switched systems is considered in [73], while for continuous case in [70]. Problem of fault detection and isolation for continuous-time switched systems using H_-/H_∞ is reported in [94]. Fault detection (FD) problem of uncertain discrete-time switched systems under the constrained switching law and external disturbances is addressed in [90]. Adaptive threshold was set, based on the bounds of disturbances and the individual performance index for each subsystem. Most of the studies for the problems have assumed that the fault detection filter is switching with the subsystems in synchronous manner. In this way, the problem is reduced to multiple linear dynamical systems simply. Although the asynchronous problem has gained attention of FD research community in recent years [89, 73, 93], still it deserves more attention to be paid. In [89], a maximum dwell time technique is employed to deal with the asynchronous switching control problem. In [73] fault detection problem for discrete switched systems was studied with the assumption that the governing switching signal is unknown. In [93], problem of fault detection, for continuous-time switched systems, under asynchronous switching, was addressed.

The rest of the chapter is organized as follows: Next, research contribution of this thesis in the area of fault detection and isolation for switched systems, is presented. After that, in Section 3.2, a scheme for fault detection is proposed, while in Section 3.3, fault detection and isolation is addressed. Next in Section 3.4, fault detection and isolation in uncertain switched system is presented, followed by chapter summary in Section 3.5.

Research Contribution

Particularly, [93] is one of the motivating works, to explore further the asynchronous switching problem in fault diagnosis. In the aforementioned work H_∞ filtering technique is used to formulate and design the filter structure. In our opinion, in that design technique it is hard to solve numerically the linear constraint equation and linear matrix inequalities (LMIs), simultaneously (remark 2 in [93]). Further, only the H_∞ norm is utilized in there.

In this chapter, we present schemes for FD, and FDI of continuous-time switched systems while taking into account the case of asynchronous switching between filters and subsystems. The switching phenomenon assumes average dwell time (ADT) constraints. The proposed schemes have the following outstanding features. First, the attention is given to the optimal solution in the sense that the effect of unknown inputs on residual signal is minimized, whereas that of the fault is maximized simultaneously. To this end, H_-/H_∞ optimization index is utilized for the problem of FDI under asynchronously switched systems. Second, the solution of the proposed FDFs are formulated in the form of LMIs which are computationally more tractable. Third, fault isolation capability is achieved in straightforward way along with fault detection, while investigating the complex asynchronous problem of switched systems.

3.2 Fault Detection in Switched Systems

In this section, we present a solution to the fault detection problem of continuous-time switched systems, while taking into account the case of asynchronous switching between filters and subsystems. The switching phenomenon assumes average dwell time (ADT) constraints. Effect of unknown inputs on residual signal is minimized. To this end, problem is formulated into H_∞ filtering problem. Proposed scheme is formulated in the form of LMIs.

3.2.1 Problem Formulation: Fault Detection in Switched Systems

System Description

Continuous-time switched systems (2.8), defined in Chapter 2, is considered here,

$$\begin{aligned}\dot{x}(t) &= A_{\sigma(t)}x\left(t\right) + B_{\sigma(t)}u\left(t\right) + B_{d\sigma(t)}d\left(t\right) + B_{f\sigma(t)}f(t)\\ y(t) &= C_{\sigma(t)}x(t) + D_{\sigma(t)}u(t) + D_{d\sigma(t)}d\left(t\right) + D_{f\sigma(t)}f(t)\end{aligned} \tag{3.1}$$

In order to generate residual signal the switched fault detection filter (2.9), defined in Chapter 2, is used as residual generator.

$$\dot{\hat{x}}(t) = A_{\sigma'(t)}\hat{x}(t) + B_{\sigma'(t)}u(t) - L_{\sigma'(t)}(y(t) - \hat{y}(t))$$
$$\hat{y}(t) = C_{\sigma'(t)}\hat{x}(t) + D_{\sigma'(t)}u(t)$$
$$r(t) = H_{\sigma'(t)}(y(t) - \hat{y}(t)) \tag{3.2}$$

As per discussion in Chapter 2, for asynchronous case problem is formulated as follows.

Matched Period: During the matched period, ith subsystem and ith filter are in operation, see Fig.2.7. We augment the switched system (3.1) and detection filter (3.2) into the following compact representation during matched period

$$\dot{\tilde{x}}(t) = \tilde{A}_i\tilde{x}(t) + \tilde{B}_i\omega(t)$$
$$r(t) = \tilde{C}_i\tilde{x}(t) + \tilde{D}_i\omega(t), \tag{3.3}$$

where,

$$\tilde{x}(t) = \begin{bmatrix} x(t)^T & \hat{x(t)}^T \end{bmatrix}^T, \omega(t) = \begin{bmatrix} u(t)^T & d(t)^T \end{bmatrix}^T$$
$$\tilde{A}_i = \begin{bmatrix} A_i & 0 \\ -L_iC_i & A_i + L_iC_i \end{bmatrix}, \tilde{D}_i = \begin{bmatrix} 0 & H_iD_{di} \end{bmatrix}$$
$$\tilde{B}_i = \begin{bmatrix} B_i & B_{di} \\ B_i & -L_iD_{di} \end{bmatrix}, \tilde{C}_i = \begin{bmatrix} H_iC_i & -H_iC_i \end{bmatrix}$$

Unmatched Period: During the unmatched period, jth subsystem and ith filter are switched together, see Fig.2.7. We augment the switched system (3.1) and detection filter (3.2) into the following compact representation during unmatched period

$$\dot{\tilde{x}}(t) = \tilde{A_{ij}}\tilde{x}(t) + \tilde{B_{ij}}\omega(t)$$
$$r(t) = \tilde{C_{ij}}\tilde{x}(t) + \tilde{D_{ij}}\omega(t), \tag{3.4}$$

where,

$$\tilde{x}(t) = \begin{bmatrix} x(t)^T & \hat{x}(t)^T \end{bmatrix}^T, \omega(t) = \begin{bmatrix} u(t)^T & d(t)^T \end{bmatrix}^T$$

$$\tilde{A_{ij}} = \begin{bmatrix} A_j & 0 \\ -L_iC_j & A_i + L_iC_i \end{bmatrix}, \tilde{B_{ij}} = \begin{bmatrix} B_j & B_{dj} \\ B_i - L_iD_j + L_iD_i & -L_iD_{dj} \end{bmatrix},$$

$$\tilde{C_{ij}} = \begin{bmatrix} H_iC_j & -H_iC_i \end{bmatrix}, \tilde{D_{ij}} = \begin{bmatrix} H_iD_j - H_iD_i & H_iD_{dj} \end{bmatrix}$$

Problem 1. *Given the switched system (3.1), subject to actuator and sensor faults, under the effect of disturbances and noise, design a fault detection filter (FDF) to detect the faults, such that, the system for average dwell time (ADT) switching, under asynchronous switching is exponentially stable with* H_∞ *performance,* $\parallel G_{r\omega} \parallel_\infty < \gamma_i$*, and* $\parallel G_{r\omega} \parallel_\infty < \gamma_{ij}$.

3.2.2 Solution to the H_∞ Fault Detection Problem

H_∞ solution is proposed in the form of following Theorem 1 and algorithm.

Main Results

Theorem 1. *Suppose the residual generator (3.3) and (3.4) satisfy the assumptions A1-A3, if, there exist symmetric positive definite matrix* $P_i > 0, P_{ij} > 0$*, for* $i \neq j; i,j \in N$*, while any switching signal with ADT, /* $\tau_\alpha > {\tau_\alpha}^* = \frac{ln(\mu 1 \mu 2)}{\zeta^*}, 0 < \zeta^* < \alpha$*, then; for a given scalar* $\alpha_i \geq 0, \rho_i \geq 0, \mu_1 \geq 1, \mu_2 \geq 1, \beta_i \geq 1$,

$$\int_0^\infty (r^T)(r)dt < \gamma_i^2 \int_0^\infty (\omega^T)(\omega)dt \tag{3.5}$$

and

$$\int_0^\infty (r^T)(r)dt < \gamma_{ij}^2 \int_0^\infty (\omega^T)(\omega)dt \tag{3.6}$$

such that, the following set of LMIs has a solution;

$$\begin{bmatrix} P_{11ij} & P_{12ij} \\ * & P_{22ij} \end{bmatrix} > 0 \quad , \begin{bmatrix} P_{11i} & P_{12i} \\ * & P_{22i} \end{bmatrix} > 0 \tag{3.7}$$

$$\begin{bmatrix} P_{11j} & P_{12j} \\ * & P_{22j} \end{bmatrix} \leq \mu_1 \begin{bmatrix} P_{11ij} & P_{12ij} \\ * & P_{22ij} \end{bmatrix} \tag{3.8}$$

$$\begin{bmatrix} P_{11ij} & P_{12ij} \\ * & P_{22ij} \end{bmatrix} \leq \mu_2 \begin{bmatrix} P_{11i} & P_{12i} \\ * & P_{22i} \end{bmatrix} \tag{3.9}$$

$$\begin{bmatrix} \Psi_{11} & \Psi_{12} & P_{11i}B_i + P_{12i}B_i & \Psi_{14} & \Psi_{15} \\ * & \Psi_{22} & {P_{12i}}^T B_i + P_{22i}B_i & \Psi_{24} & \Psi_{25} \\ * & * & -{\gamma_i}^2 I & 0 & 0 \\ * & * & * & -{\gamma_i}^2 I & \Psi_{45} \\ * & * & * & * & -I \end{bmatrix} < 0 \tag{3.10}$$

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & \Omega_{15} \\ * & \Omega_{22} & \Omega_{23} & \Omega_{24} & \Omega_{25} \\ * & * & -{\gamma_{ij}}^2 I & 0 & \Omega_{35} \\ * & * & * & -{\gamma_{ij}}^2 I & \Omega_{45} \\ * & * & * & * & -I \end{bmatrix} < 0 \tag{3.11}$$

where,

$\Psi_{11} = {A_i}^T P_{11i} + P_{11i}A_i + \alpha_i P_{11i} - C_i^T L_i^T P_{12i}^T - P_{12i}L_iC_i$

$\Psi_{12} = {A_i}^T P_{12i} + P_{12i}A_i + \alpha_i P_{12i} + P_{12i}L_iC_i - C_i^T L_i^T P_{22i},$

$\Psi_{14} = P_{11i}B_{di} - P_{12i}L_iD_{di}, \Psi_{15} = {C_i}^T {H_i}^T,$

$\Psi_{22} = {A_i}^T P_{22i} + \alpha_i P_{22i} + {C_i}^T L_i^T P_{22i} + P_{22i}A_i + P_{22i}L_iC_i$

$\Psi_{24} = {P_{12i}}^T B_{di} - P_{22i}L_iD_{di}, \Psi_{25} = -{C_i}^T {H_i}^T,\ \Psi_{45} = {D_{di}}^T H_i^T$

$\Omega_{11} = {A_j}^T P_{11ij} + P_{11ij}A_j - C_j^T L_i^T P_{12ij}^T - \rho_i P_{11ij} - P_{12ij}L_iC_j$

$\Omega_{12} = {A_j}^T P_{12ij} + P_{12ij}A_i - \rho_i P_{12ij} + P_{12ij}L_iC_i - C_j^T L_i^T P_{22ij}$

$\Omega_{13} = P_{11ij}B_j + P_{12ij}B_i - P_{12ij}L_iD_j + P_{12ij}L_iD_i$

$\Omega_{14} = P_{11ij}B_{dj} - P_{12ij}B_{fi}L_iD_{dj}, \Omega_{15} = {C_j}^T H_i^T$

$\Omega_{22} = {A_i}^T P_{22ij} - \rho_i P_{22ij} + {C_i}^T L_i^T P_{22ij} + P_{22ij}A_i + P_{22ij}L_iC_i$

$\Omega_{23} = P_{ij}^T B_j + P_{22ij}B_i - P_{22ij}L_iD_j + P_{22ij}L_iD_i,\ \Omega_{24} = {P_{12ij}}^T B_{dj} - P_{22ij}L_iD_{dj},$

$\Omega_{25} = -{C_i}^T {H_i}^T, \Omega_{35} = D_j^T H_i^T - D_i^T H_i^T, \Omega_{45} = {D_{dj}}^T H_i^T$

Proof. For the desired L_i and H_i our strategy is to find out H_∞ norm of $F_iG_{di}, \forall i \in \{1, 2, ...N\}$ while considering matched and unmatched periods as follows.

Matched Period: We consider augmented system (3.3), in this duration. Using Lemma 3 (Appendix),

$$\dot{V}_i(\tilde{x}(t)) \leq -\alpha_i V_i(\tilde{x}(t)) - r^T r(t) + \gamma_i^2 \omega(t)^T \omega(t) \tag{3.12}$$

Considering the following Lyapunov function,

$$V_i(\tilde{x}(t)) = \tilde{x}^T(t) P_i \tilde{x}(t) \tag{3.13}$$

Differentiating (3.13) along the trajectory of (3.3)

$$\dot{V}_i(\tilde{x}(t)) = \dot{\tilde{x}}^T(t) P_i \tilde{x}(t) + \tilde{x}^T(t) P_i \dot{\tilde{x}}(t). \tag{3.14}$$

Substituting (3.13), (3.14), and $r(t)$ from (3.3) in (3.12),

$$\begin{aligned} &\dot{\tilde{x}}^T(t) P_i \tilde{x}(t) + \tilde{x}^T(t) P_i \dot{\tilde{x}}(t) + \left[\tilde{C}_i \tilde{x}(t) + \tilde{D}_i \omega(t)\right]^T \left[\tilde{C}_i \tilde{x}(t) + \tilde{D}_i \omega(t)\right] \\ &\leq -\alpha_i \tilde{x}^T(t) P_i \tilde{x}(t) + \gamma_i^2 \omega(t)^T \omega(t). \end{aligned} \tag{3.15}$$

Substituting the expression for $\dot{\tilde{x}}(t)$ from (3.3) in (3.15),

$$\begin{aligned} &\left[\tilde{A}_i \tilde{x}(t) + \tilde{B}_i \omega(t)\right]^T P_i \tilde{x}(t) + \tilde{x}^T(t) P_i \left[\tilde{A}_i \tilde{x}(t) + \tilde{B}_i \omega(t)\right] \\ &+ (\tilde{x}^T(t) \tilde{C}_i^T + \omega^T(t) \tilde{D}_i^T)(\tilde{C}_i \tilde{x}(t) + \tilde{D}_i \omega(t)) \\ &\leq -\alpha_i \tilde{x}^T(t) P_i \tilde{x}(t) + \gamma_i^2 \omega(t)^T \omega(t)). \end{aligned} \tag{3.16}$$

(3.16) can be written easily in following form,

$$\begin{aligned} &\left[\tilde{x}^T(t) \tilde{A}_i^T P_i + \omega^T(t) \tilde{B}_i^T P_i\right] \tilde{x}(t) + \tilde{x}^T(t) P_i \tilde{A}_i \tilde{x}(t) + \tilde{x}^T(t) P_i \tilde{B}_i \omega(t) + \tilde{x}^T(t) \tilde{C}_i^T \tilde{C}_i \tilde{x}(t) \\ &+ \tilde{x}^T(t) \tilde{C}_i^T \tilde{D}_i \omega(t) + \omega^T(t) \tilde{D}_i^T \tilde{C}_i \tilde{x}(t) + \omega^T(t) \tilde{D}_i^T \tilde{D}_i \omega(t)) + \alpha_i \tilde{x}^T(t) P_i \tilde{x}(t) \\ &- {\gamma_i}^2 \omega^T(t) \omega(t) \leq 0. \end{aligned} \tag{3.17}$$

Further, the above inequality can be written as

$$\begin{bmatrix} \tilde{x}^T(t) & \omega^T(t) \end{bmatrix} M \begin{bmatrix} \tilde{x}(t) \\ \omega(t) \end{bmatrix} \leq 0, \tag{3.18}$$

where,

$$M = \begin{bmatrix} \tilde{A}_i^T P_i + P_i \tilde{A}_i + \tilde{C}_i^T \tilde{C}_i + \alpha_i P_i & P_i \tilde{B}_i + \tilde{C}_i^T \tilde{D}_i \\ * & \tilde{D}_i^T \tilde{D}_i - \gamma_i^2 I \end{bmatrix}.$$

For (3.18) to hold, it is required that

$$M < 0 \tag{3.19}$$

After Schur's compliment is applied to (3.19), we get

$$\begin{bmatrix} \tilde{A}_i^T P_i + P_i \tilde{A}_i + \alpha_i P_i & P_i \tilde{B}_i & \tilde{C}_i^T \\ * & -\gamma_i^2 I & \tilde{D}_i^T \\ * & * & -I \end{bmatrix} < 0. \tag{3.20}$$

Next, by substituting augmented system,(3.3), in (3.20), LMI (3.10) is obtained. Notice that, LMIs (3.7)-(3.9) are general requirements for model-based FD in asynchronous switching paradigm [93].

Unmatched Period: We consider augmented system (3.4), in this duration. Using Lemma 3,

$$\dot{V}_{ij}(\tilde{x}(t)) \leq \rho_i V_{ij}(\tilde{x}(t)) - r(t)^T r(t) + \gamma_{ij}^2 \omega(t)^T \omega(t) \tag{3.21}$$

where $i \neq j$ and $i, j \in N$. Considering the following Lyapunov function,

$$V_{ij}(\tilde{x}(t)) = \tilde{x}^T(t) P_{ij} \tilde{x}(t) \tag{3.22}$$

Differentiating (3.22) along the trajectory of (3.4),

$$\dot{V}_{ij}(\tilde{x}(t)) = \dot{\tilde{x}}^T(t) P_{ij} \tilde{x}(t) + \tilde{x}^T(t) P_{ij} \dot{\tilde{x}}(t). \tag{3.23}$$

Substituting (3.22), (3.23), and $r(t)$ from (3.4) in (3.21),

$$\begin{aligned}&\dot{\tilde{x}}^T(t)P_{ij}\tilde{x}(t) + \tilde{x}^T(t)P_{ij}\dot{\tilde{x}}(t) + \left[\tilde{C}_{ij}\tilde{x}(t) + \tilde{D}_{ij}\omega(t)\right]^T \left[\tilde{C}_{ij}\tilde{x}(t) + \tilde{D}_{ij}\omega(t)\right] \\ &\leq +\rho_i\tilde{x}^T(t)P_{ij}\tilde{x}(t) + \gamma_{ij}^2\omega(t)^T\omega(t).\end{aligned} \tag{3.24}$$

Substituting the expression for $\dot{\tilde{x}}(t)$ from (3.4) in (3.24),

$$\begin{aligned}&\left[\tilde{A}_{ij}\tilde{x}(t) + \tilde{B}_{ij}\omega(t)\right]^T P_{ij}\tilde{x}(t) + \tilde{x}^T(t)P_{ij}\left[\tilde{A}_{ij}\tilde{x}(t) + \tilde{B}_{ij}\omega(t)\right] \\ &+ (\tilde{x}^T(t)\tilde{C}_{ij}^T + \omega^T(t)\tilde{D}_{ij}^T)(\tilde{C}_{ij}\tilde{x}(t) + \tilde{D}_{ij}\omega(t)) \\ &\leq +\rho_{ij}\tilde{x}^T(t)P_{ij}\tilde{x}(t) + \gamma_{ij}^2\omega(t)^T\omega(t)).\end{aligned} \tag{3.25}$$

(3.25) can be written easily in following form

$$\begin{aligned}&\left[\tilde{x}^T(t)\tilde{A}_{ij}^TP_{ij} + \omega^T(t)\tilde{B}_{ij}^TP_{ij}\right]\tilde{x}(t) + \tilde{x}^T(t)P_{ij}\tilde{A}_{ij}\tilde{x}(t) + \tilde{x}^T(t)P_{ij}\tilde{B}_{ij}\omega(t) \\ &+ \tilde{x}^T(t)\tilde{C}_{ij}^T\tilde{C}_{ij}\tilde{x}(t) + \tilde{x}^T(t)\tilde{C}_{ij}^T\tilde{D}_{ij}\omega(t) + \omega^T(t)\tilde{D}_{ij}^T\tilde{C}_{ij}\tilde{x}(t) + \omega^T(t)\tilde{D}_{ij}^T\tilde{D}_{ij}\omega(t)) \\ &- \rho_{ij}\tilde{x}^T(t)P_{ij}\tilde{x}(t) - \gamma_{ij}{}^2\omega^T(t)\omega(t) \leq 0\end{aligned} \tag{3.26}$$

Further, the above inequality can be written as,

$$\begin{bmatrix}\tilde{x}^T(t) & \omega^T(t)\end{bmatrix} M \begin{bmatrix}\tilde{x}(t) \\ \omega(t)\end{bmatrix} \leq 0. \tag{3.27}$$

where,

$$M = \begin{bmatrix}\tilde{A_{ij}}^TP_{ij} + P_{ij}\tilde{A}_{ij} + \tilde{C_{ij}}^T\tilde{C}_{ij} - \rho_iP_{ij} & P_{ij}\tilde{B}_{ij} + \tilde{C_{ij}}^T\tilde{D}_{ij} \\ * & \tilde{D_{ij}}^T\tilde{D}_{ij} - \gamma_{ij}^2I\end{bmatrix}$$

For (3.27) to hold, it is required that,

$$M < 0 \tag{3.28}$$

After Schur's compliment is applied to (3.19), we get

$$\begin{bmatrix}\tilde{A_{ij}}^TP_{ij} + P_{ij}\tilde{A}_{ij} - \rho_iP_{ij} & P_{ij}\tilde{B}_{ij} & \tilde{C_{ij}}^T \\ * & -\gamma_{ij}^2I & \tilde{D_{ij}}^T \\ * & * & -I\end{bmatrix} < 0. \tag{3.29}$$

Next, by substituting augmented system,(3.4), in (3.29), LMI (3.11) is obtained. Now, the proof is completed which ensures disturbance attenuation level $\gamma_i, \forall i \in \{1, 2, ...N\}$ and fault detection capability achieved by the derived filter. □

In the next subsection, we present an algorithm, which is based on the results derived in Theorem 1, in stepwise simplified form to design our objective filter.

3.2.3 Algorithm

Let the model of the switched system is given as in 3.1. By taking into account the assumptions, in Chapter 2,

1. Check the detectability of all subsystems (modes), i.e., whether (A_i, C_i) is detectable $\forall i \in \{1, 2, ...N\}$, if yes then proceed to next step, if no, then it is not possible to proceed

2. Set the ADT parameters, μ_1, μ_2, α_i, and ρ_i, $\forall \quad i \in \{1, 2, ...N\}$, then solve the LMIs (3.7)-(3.11) simultaneously to get the optimal values of γ_i, and γ_{ij} $\forall \quad i \in \{1, 2, ...N\}$.

3. Find filter parameters L_i and H_i $\forall \quad for\ i \neq j$, and $i, j \in N$ from Step 2, as well.

3.2.4 Threshold Computation and Residual Evaluation

After successful residual generation, the next step is to evaluate the residual signal. The importance of this step is due to the fact that residual may be nonzero even if there is no fault in the system. In this work following residual evaluation function, which is based on RMS energy of the residual signal, is utilized.

$$J_{RMS} = \parallel r(t) \parallel_{RMS} = (\frac{1}{T}\int_{t}^{t+T} \parallel r(\tau) \parallel^2 d\tau)^{\frac{1}{2}} \tag{3.30}$$

where, T is the evaluation window.

Along with residual evaluation function, the threshold computation is also required for efficient detection of faults. Threshold value is the maximum influence of unknown inputs (disturbances, noises) and model uncertainties on the residual signal in the absence of faults. Threshold can also be of different types like fixed, adaptive, or dynamic [5, 95], depending on application under consideration. In this research the following threshold is employed, which is defined as,

$$J_{th,RMS,2} = \sup_{\|\omega(t)\|_2 \leq \delta_{d,2}+\delta_{u,2}, f(t)=0} J_{RMS}, \tag{3.31}$$

and computed as,

$$J_{th,RMS,2} = \frac{\gamma_i^*(\delta_{d,2} + \delta_{u,2})}{\sqrt{T}} \tag{3.32}$$

where $\gamma_i^* = min(\gamma_i)$, $\delta_{d,2}$ denotes the set of L_2-norm bounded disturbance signals and $\delta_{u,2}$ denotes the set of L_2-norm bounded input signals. Finally, decision about the presence of fault in the system is made by the following logic

- $J_{RMS} \leq J_{th,RMS,2} \Longrightarrow$ No FAULT
- $J_{RMS} > J_{th,RMS,2} \Longrightarrow$ Detected FAULT

3.2.5 Case Study: Battery Converter Unit of Hybrid Electric Vehicle

In this section, case study, battery converter unit of hybrid electric vehicle, is introduced, which is considered for applying proposed results in the section.

Battery Converter Unit of Hybrid Electric Vehicle

In hybrid electric vehicles there are more electrical components than conventional vehicles, such as ultra capacitors, sensors, power electronics converters, electrical machines, micro controllers, and processors. For this reason it is a "hybrid" design, in electrical and mechanical domains. Typical components used in HEVs are depicted in Fig. 3.1.

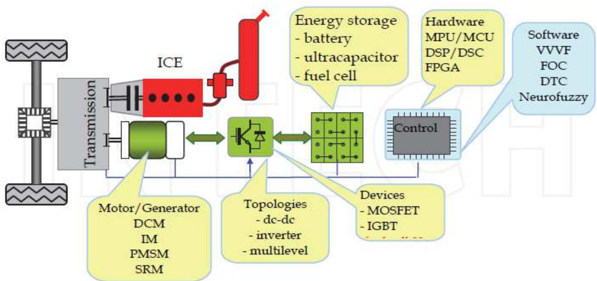

Figure 3.1: Main components of a hybrid electric vehicle [2]

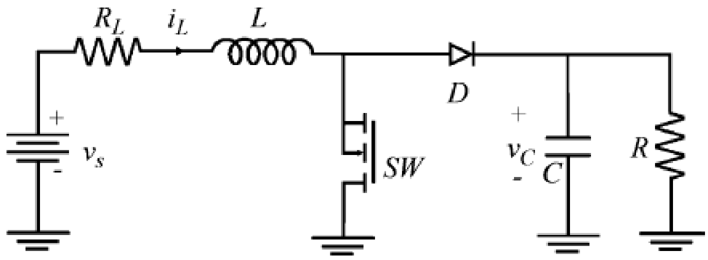

Figure 3.3: Battery-converter unit [3]

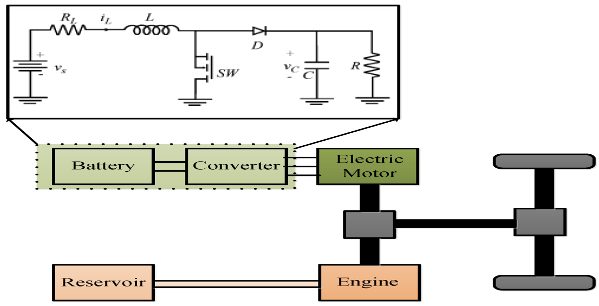

Figure 3.2: Main components of a hybrid electric vehicle [2]

Drive Train and Battery-Converter Unit

Drive train is an energy sourcing strategy implemented in HEVs. There are different drive train configurations being employed in production of HEVs: series HEV, parallel HEV, series-parallel HEV, and complex HEV that have their own pros and cones, see [96] for details. In a typical parallel design, consisting of an ICE and an electric motor in a torque-combining configuration, either the ICE or the electric motor can be considered the primary energy source depending on the vehicle design and energy management strategy. The drive train can also be designed so that the ICE and electric motor are both responsible for propulsion or each is the prime mover at a certain time in the drive cycle, see Fig. 3.2. In electric line of drive train dc-dc converter is a major part. The battery converter unit is depicted in Fig. 3.3. We assume that both state variables i_L (inductor current) and v_C (capacitor voltage) are available for measurement, and voltage v_s is known. The battery converter unit is a switched system, operating in two modes. In mode 1, the transistor switch SW is CLOSED and the diode switch D is OPENED. In mode 2, the transistor switch SW is OPENED and the diode switch D is CLOSED. System matrices in mode 1 and mode 2 take the form

$$A_1 = \begin{bmatrix} -\frac{R_l}{L} & 0 \\ 0 & -\frac{1}{RC} \end{bmatrix}, B_1 = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -\frac{R_L}{L} & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix}, B_2 = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}$$

One of possible faults in the battery converter unit is a "biased voltage" in capacitor of converter circuit, which occurs due to degradation of capacitor with time [3]. The capacitance $C(t)$ can be given as $C(t) = C + \lambda_C(t)$, where, C is the nominal capacitance, and $\lambda_C(t)$ describes the fault magnitude. Other possible fault is related to battery which is of "intermittent type", that is, there are sudden dips in available output voltage to the converter. This malfunction can be due to frequent stop-and-go or acceleration-and-deceleration vehicle operation. In this situation, the battery will rapidly charge and discharge and heat up [97].

Results and Discussion

For the case study, simulation time is setup for 30 sec, such that, subsystem 1 is activated when $\sigma(t) = 0$ and subsystem 2 when $\sigma(t) = 1$. Based on our proposed FD approach we are able to detect two faults. Fault $f_1(t)$ is considered as battery fault, while $f_2(t)$ as converter fault in capacitor. Implications of $f_1(t)$ on the operation of engine of HEV is the fluctuations in speed and torque. Consequence of $f_2(t)$ is the constant drop in speed and torque of the engine from reference points. Details of temporal switching behaviour of subsystems, filters and faults can be seen in Fig. (3.4). Switching signal $\sigma(t)$ is applied according to ADT value of 1.6894 for parameters $\mu_1 = 1.5, \mu_2 = 1.5, \alpha_i = 0.5, \zeta^* = 0.48$ according to definition of ADT, in Chapter 2. Further, for simulation study we take the disturbance signal as L_2 norm bounded with $\delta_{d,2} \leq 1$ for each mode. In practice noise signal is of stochastic nature, for simplification, here we assume the noise signal of deterministic nature for each mode. Next, we discuss the results that are based on the following design parameters of boost converter [3]: $R_L = 0.2$ ohms, $L = 0.05$ mH, C =

200 μ F, and $R = 24$ ohms. Dynamics of boost converter are given below.

$$A_1 = \begin{bmatrix} -4000 & 0 \\ 0 & -208.33 \end{bmatrix}, B_1 = \begin{bmatrix} 20000 \\ 0 \end{bmatrix}, C_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$B_{d1} = \begin{bmatrix} -0.1 & 0.03 \\ -0.2 & 0.1 \end{bmatrix}, D_{d1} = \begin{bmatrix} 0.02 & -0.1 \\ -0.01 & 0.02 \end{bmatrix}, B_{f1} = \begin{bmatrix} 20000 & 0 \\ 0 & 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -4000 & -20000 \\ 5000 & -208.33 \end{bmatrix}, B_2 = \begin{bmatrix} 20000 \\ 0 \end{bmatrix}, C_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$B_{d2} = \begin{bmatrix} -0.01 & -0.03 \\ 0.1 & -0.16 \end{bmatrix}, D_{d2} = \begin{bmatrix} 0.11 & 0.3 \\ 0.2 & -0.01 \end{bmatrix}, B_{f2} = \begin{bmatrix} 20000 & 0 \\ 0 & 0 \end{bmatrix},$$

$$D_{f1} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, D_{f2} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix},$$

Next, during fault simulation, filter parameters are found to be:

$$L_1 = \begin{bmatrix} -40000 & 0 \\ 1 & -1 \end{bmatrix}, H_1 = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}, L_2 = \begin{bmatrix} -22222 & 0 \\ 0 & -1 \end{bmatrix}, H_2 = \begin{bmatrix} 1.11 & 0 \\ 0 & 0 \end{bmatrix}$$

Then, the BCU is simulated according to Fig. (3.4), therein temporal status of switching signals for subsystems and filters, battery fault, and capacitor fault is depicted. Notice that there are unmatched periods from 0-1, 6-7, 15-16, and 21-22 respectively. Matched periods are from 1-6, 7-15, 16-21, and 22-30 respectively. Battery fault occurs for short time as a pulse type during Mode 1 at 3 sec, while capacitor fault occurs during Mode 2 at 8 sec as step type. Fig. (3.5) shows the residual signals when none of the fault occurs. The problem of false fault detection can be observed there. It is due to the nature of asynchronous switching, that during unmatched period, residual is non-zero without any fault. To overcome this difficulty, residuals are evaluated according to (3.30) and threshold is computed as per (3.32). Threshold during unmatched period, Mode 1 and filter 2, is $th_{21} = 6.2154$. During unmatched period, Mode 2 and filter 1, is $th_{12} = 4.6554$. Similarly during matched period, Mode 1 and filter 1, is $th_1 = 1.0128$ and during matched period, Mode 2 and filter 2, is $th_2 = 1.2650$. Again the results are studied, in case of none of the faults occurs, but employing residual evaluation and threshold setting, shown in (3.6). It is easy to see that problem of false fault detection is resolved. The evaluated residuals during matched as well as unmatched period are less than the thresholds during the respective durations. After that, on the occurrence of faults, evaluated residual signals and computed thresholds are depicted in Fig. (3.7) and Fig. (3.8). Both faults are detected successfully, when evaluated residuals cross the thresholds and further no any false alarm is generated during unmatched period.

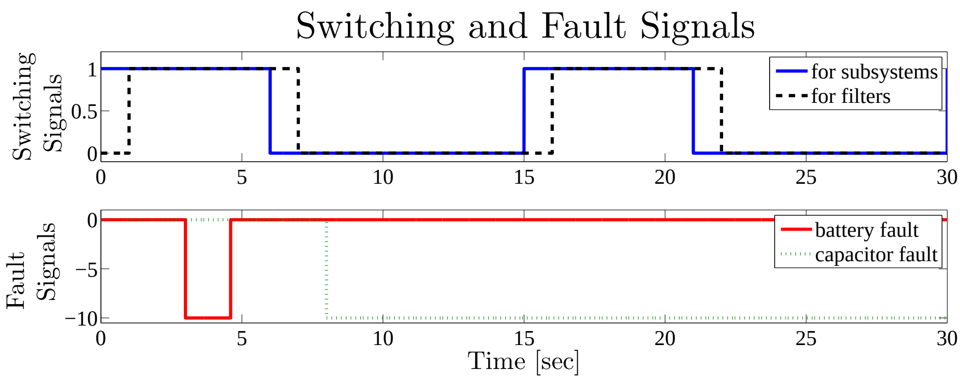

Figure 3.4: Switching signals: for subsystems $\sigma(t)$; for filters $\sigma'(t)$, Fault signals: of battery fault $f_1(t)$; of capacitor fault $f_2(t)$

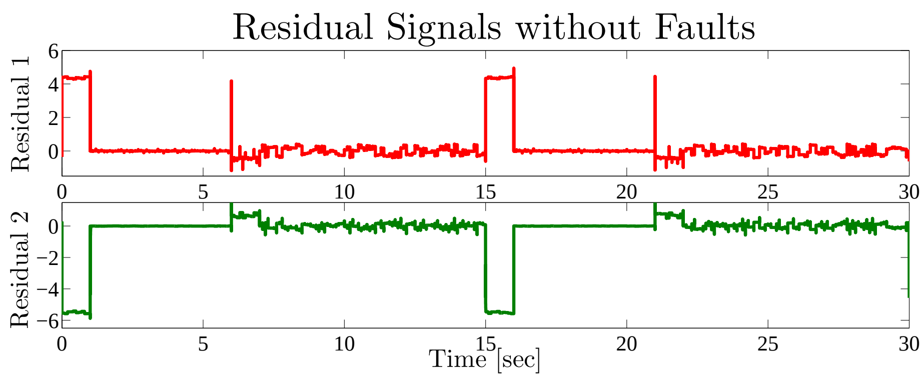

Figure 3.5: Residual 1, $r_1(t)$, and Residual 2, $r_2(t)$ without any fault

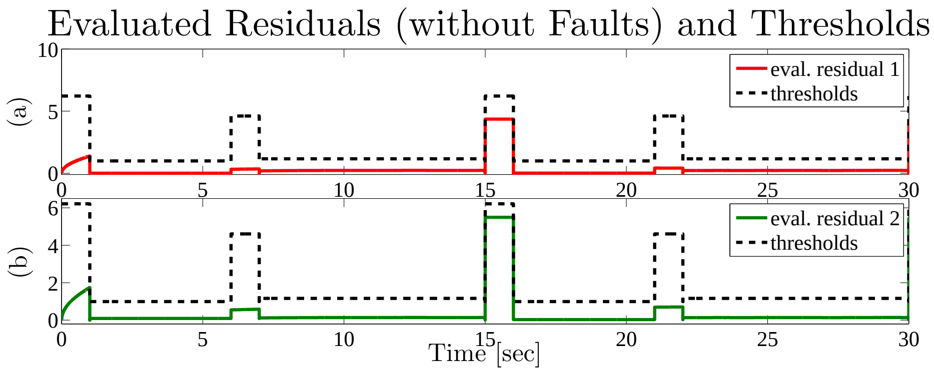

Figure 3.6: (a) Evaluated residual 1 without any fault, $er_1(t)$, and thresholds in both modes: (b) Evaluated residual 2 without any fault, $er_2(t)$, and thresholds in both modes

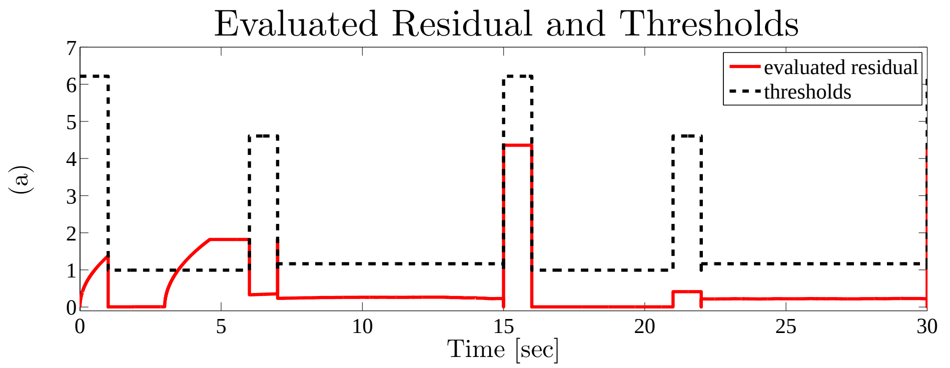

Figure 3.7: (a) Evaluated residual when battery fault occurring, $er_1(t)$, and thresholds in both modes

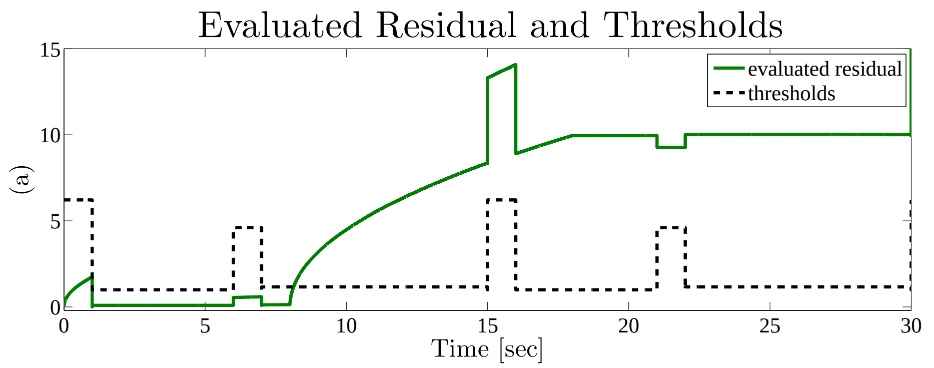

Figure 3.8: (a) Evaluated residual when capacitor fault occurring, $er_1(t)$, and thresholds in both modes

In the following section, problem of fault detection is extended to fault detection and isolation for switched systems under asynchronous paradigm.

3.3 Fault Detection and Isolation in Switched Systems

In this section, we present a solution to the fault detection and isolation problem of continuous-time switched systems while taking into account the case of asynchronous switching between filters and subsystems. The switching phenomenon assumes average dwell time (ADT) constraints. The proposed solution has the following outstanding features. First, the attention is given to the optimal solution in the sense that the effect of unknown inputs on residual signal is minimized whereas that of the fault is maximized simultaneously. To this end, H_-/H_∞ optimization index is utilized for the problem of asynchronously switched systems. Second, the solution of the proposed FDF is formulated in the form of LMIs which are computationally more tractable than that in [93]. Third, and a significant contribution of proposed work is to achieve the fault isolation capability in straightforward way along with fault detection while investigating the complex asynchronous problem of switched systems.

3.3.1 Problem Formulation: Fault Detection and Isolation in Switched Systems

System Description

Consider the continuous-time switched systems, (2.8) defined in Chapter 2,

$$\begin{aligned} \dot{x}(t) &= A_{\sigma(t)}x\left(t\right) + B_{\sigma(t)}u\left(t\right) + B_{d\sigma(t)}d\left(t\right) + B_{f\sigma(t)}f(t) \\ y(t) &= C_{\sigma(t)}x(t) + D_{\sigma(t)}u(t) + D_{d\sigma(t)}d\left(t\right) + D_{f\sigma(t)}f(t) \end{aligned} \tag{3.33}$$

In order to generate residual signal the switched fault detection filter (2.9), defined in Chapter 2, is used as residual generator.

$$\begin{aligned} \dot{\hat{x}}(t) &= A_{\sigma'(t)}\hat{x}(t) + B_{\sigma'(t)}u(t) - L_{\sigma'(t)}(y(t) - \hat{y}(t)) \\ \hat{y}(t) &= C_{\sigma'(t)}\hat{x}(t) + D_{\sigma'(t)}u(t) \\ r(t) &= H_{\sigma'(t)}(y(t) - \hat{y}(t)) \end{aligned} \tag{3.34}$$

As per discussion in Chapter 2, for asynchronous case, problem is formulated as follows.

Matched Period

During the matched period, *ith* subsystem and *ith* filter are in operation, see Fig. 2.7. We augment the switched system (3.33) and fault detection filter (3.34) into the following compact representation

$$\begin{aligned}\dot{\tilde{x}}(t) &= \tilde{A}_i\tilde{x}(t) + \tilde{B}_i\omega(t)\\ r(t) &= \tilde{C}_i\tilde{x}(t) + \tilde{D}_i\omega(t)\end{aligned} \tag{3.35}$$

where,

$$\tilde{x}(t) = \begin{bmatrix} x(t)^T & \hat{x(t)}^T \end{bmatrix}^T, \omega(t) = \begin{bmatrix} u(t)^T & d(t)^T \end{bmatrix}^T$$

and

$$\tilde{A}_i = \begin{bmatrix} A_i & 0 \\ -L_iC_i & A_i + L_iC_i \end{bmatrix}, \tilde{B}_i = \begin{bmatrix} B_i & B_{di} \\ B_i & -L_iD_{di} \end{bmatrix}$$
$$\tilde{C}_i = \begin{bmatrix} H_iC_i & -H_iC_i \end{bmatrix}, \tilde{D}_i = \begin{bmatrix} 0 & H_iD_{di} \end{bmatrix}$$

Unmatched Period

During the unmatched period, *jth* subsystem and *ith* filter are in operation, see Fig. 2.7. We augment the switched system (3.33) and fault detection filter (3.34) as follows

$$\begin{aligned}\dot{\tilde{x}}(t) &= \tilde{A_{ij}}\tilde{x}(t) + \tilde{B_{ij}}\omega(t)\\ r(t) &= \tilde{C_{ij}}\tilde{x}(t) + \tilde{D_{ij}}\omega(t)\end{aligned} \tag{3.36}$$

where,

$$\tilde{x}(t) = \begin{bmatrix} x(t)^T & \hat{x}(t)^T \end{bmatrix}^T, \omega(t) = \begin{bmatrix} u(t)^T & d(t)^T \end{bmatrix}^T$$

and

$$\tilde{A_{ij}} = \begin{bmatrix} A_j & 0 \\ -L_iC_j & A_i + L_iC_i \end{bmatrix}, \tilde{B_{ij}} = \begin{bmatrix} B_j & B_{dj} \\ B_i - L_iD_j + L_iD_i & -L_iD_{dj} \end{bmatrix}$$
$$\tilde{C_{ij}} = \begin{bmatrix} H_iC_j & -H_iC_i \end{bmatrix}, \tilde{D_{ij}} = \begin{bmatrix} H_iD_j - H_iD_i & H_iD_{dj} \end{bmatrix}$$

H_-/H_∞ Index and Fault Isolation

Each subsystem of the considered switched system is linear time invariant (LTI) and therefore each subsystem individually can be represented by a transfer function $G_i(s) = (A_i, B_i, C_i, D_i)$ $\forall i \in \{1, 2, ..., N\}$. It is intended to design FDF such that there is,

1. maximum effect of fault on the residual signal, that is
   $\| r_f(t) \|_2 \geq \beta_i \| f(t) \|_2 \ \ \forall i \in \{1, 2, ...N\}$ and

2. minimum effect of unknown inputs on the residual signal that is
   $\| r_d(t) \|_2 \leq \gamma_i \| d(t) \|_2 \ \ \forall i \in \{1, 2, ...N\}$.
   Where r_f is the effect of fault on residual, and r_d is the effect of disturbance on residual signal.

Residual signal $R(s)$ during each mode is given as

$$R(s) = F_i(s)(G_{di}(s)d(s) + G_{fi}(s)f(s))$$

where, $F_i(s) = (A_i + L_iC_i, L_i, H_iC_i, H_i)$ is the post filter for each subsystem of the switched system, and and

$$G_{di}(s) = (A_i, B_{di}, C_i, D_{di})$$
$$G_{fi}(s) = (A_i, B_{fi}, C_i, D_{fi}),$$

are the transfer functions from disturbance to residual and fault to residual, respectively. Further, note that

$$F_i(s)G_{fi}(s) = (A_i + L_iC_i, B_{fi} + L_iD_{fi}, H_iC_i, H_iD_{fi}) \in \mathbf{R}H_\infty^{q\times m}$$

The above stated H_-/H_∞ objective is achievable, if a filter $F_i(s)$ can be designed for each mode by finding L_i and H_i, such that, γ_i is minimized and simultaneously ensuring $\beta_i \geq 1$ under the ADT constraint. In addition, design of filter should be such that, to achieve the fault isolation, every fault should exclusively influence a residual signal [5]. The above problem can be presented in short as following

$$\gamma_i = inf\|F_i(s)G_{di}(s)\|_\infty s.t. \| F_i(s)G_{fi}(s) \|_- \geq \beta_i,$$

Problem 2. *Given a switched system (3.33) subjected to disturbances and noises while assuming that the switching signal satisfies ADT. Design an FDF such that generated residual is robust against disturbances and noises in the sense of* H_∞*, that is,* $\| G_{r\omega} \|_\infty < \gamma_i, \| G_{r\omega} \|_\infty < \gamma_{ij}$ *and sensitive to minimum possible fault in the sense of* H_-*, that is,* $\| G_{r\omega} \|_- > \beta_i$*. In addition, the designed FDF should be such that the actuator and sensor faults can be detected and isolated with the same designed parameters.*

After the problem has been formulated, in next section we discuss the solution to H_-/H_∞ and fault isolation problem. H_-/H_∞ solution is a sort of compromise between maximizing sensitivity level and minimizing disturbance attenuation level. To this end, different variants may exist. In this research, compromise between sensitivity and attenuation level is proposed in such a way that isolation is also possible. Details of the derived results are given in following Theorem 2.

3.3.2 H_-/H_∞ based FDI Solution

Proposed solution is a sort of compromise between maximizing fault sensitivity level and minimizing disturbance attenuation level. To this end, different variants exist. In this research, compromise between sensitivity and attenuation level is proposed, such that, fault isolation is also possible. It is proposed by setting H_- index, β_i, and then optimize (minimize) the H_∞ norm, $\gamma_{\mathbf{i}}$. Conditions for matched and unmatched period are proposed in Theorem 2.

3.3.3 Main Results

Theorem 2. *Suppose the residual generator (3.35) and (3.36) satisfy the assumptions A1-A4, if there exist symmetric positive definite matrix* $P_i > 0, P_{ij} > 0$*, for* $i \neq j; i, j \in N$*, then, for a given scalar* $\alpha_i \geq 0, \rho_i \geq 0, \mu_1 \geq 1, \mu_2 \geq 1, \beta_i \geq 1$*, while any switching signal with ADT* $\tau_\alpha > {\tau_\alpha}^* = \frac{ln(\mu_1\mu_2)}{\zeta^*}, 0 < \zeta^* < \alpha$*,*

$$\int_0^\infty (r^T)(r)dt < \gamma_i^2 \int_0^\infty (\omega^T)(\omega)dt \tag{3.37}$$

and

$$\int_0^\infty (r^T)(r)dt < \gamma_{ij}^2 \int_0^\infty (\omega^T)(\omega)dt \tag{3.38}$$

such that the following set of LMIs has a solution;

$$\begin{bmatrix} P_{11ij} & P_{12ij} \\ * & P_{22ij} \end{bmatrix} > 0 \quad , \quad \begin{bmatrix} P_{11i} & P_{12i} \\ * & P_{22i} \end{bmatrix} > 0, \tag{3.39}$$

$$\begin{bmatrix} P_{11j} & P_{12j} \\ * & P_{22j} \end{bmatrix} \leq \mu_1 \begin{bmatrix} P_{11ij} & P_{12ij} \\ * & P_{22ij} \end{bmatrix}, \tag{3.40}$$

$$\begin{bmatrix} P_{11ij} & P_{12ij} \\ * & P_{22ij} \end{bmatrix} \leq \mu_2 \begin{bmatrix} P_{11i} & P_{12i} \\ * & P_{22i} \end{bmatrix}, \tag{3.41}$$

$$\begin{bmatrix} \Psi_{11} & \Psi_{12} & P_{11i}B_i + P_{12i}B_i & \Psi_{14} & \Psi_{15} \\ * & \Psi_{22} & {P_{12i}}^T B_i + P_{22i}B_i & \Psi_{24} & \Psi_{25} \\ * & * & -{\gamma_i}^2 I & 0 & 0 \\ * & * & * & -{\gamma_i}^2 I & \Psi_{45} \\ * & * & * & * & -I \end{bmatrix} < 0, \tag{3.42}$$

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & \Omega_{15} \\ * & \Omega_{22} & \Omega_{23} & \Omega_{24} & \Omega_{25} \\ * & * & -{\gamma_{ij}}^2 & 0 & \Omega_{35} \\ * & * & * & -{\gamma_{ij}}^2 I & \Omega_{45} \\ * & * & * & * & -I \end{bmatrix} < 0, \tag{3.43}$$

where,

$$\Psi_{11} = {A_i}^T P_{11i} + C_i^T {{D_{fi}}^{\dagger}}^T {B_{fi}}^T {P_{12i}}^T + P_{11i}A_i - C_i^T {{D_{fi}}^{\perp}}^T {W_i}^T + P_{12i}B_{fi}{D_{fi}}^{\dagger}C_i - W_i{D_{fi}}^{\perp}C_i + \alpha_i P_{11i}$$

$$\Psi_{12} = {A_i}^T P_{12i} + P_{12i}A_i + \alpha_i P_{12i} - P_{12i}B_{fi}D_{fi}^{\dagger}C_i + {C_i}^T {{D_{fi}}^{\dagger}}^T {B_{fi}}^T P_{22i} - C_i {{D_{fi}}^{\dagger}}^T {X_i}^T + W_i{D_{fi}}^{\perp}C_i$$

$$\Psi_{14} = P_{11i}B_{di} + P_{12i}B_{fi}D_{fi}^{\dagger}D_{di} - W_i D_{fi}^{\perp}D_{di}, \Psi_{15} = {C_i}^T {{D_{fi}}^{\dagger}}^T {\beta_i}^T + {C_i}^T {{D_{fi}}^{\perp}}^T {S_i}^T {\beta_i}^T$$

$$\Psi_{22} = {A_i}^T P_{22i} + \alpha_i P_{22i} - {C_i}^T {D_{fi}^{\dagger}}^T B_{fi}^T P_{22i} + P_{22i}A_i + {C_i}^T {D_{fi}^{\perp}}^T {X_i}^T - P_{22i}B_{fi}{D_{fi}}^{\dagger}C_i + X_i{D_{fi}}^{\perp}C_i, \Psi_{24} = {P_{12i}}^T B_{di} + P_{22i}B_{fi}{D_{fi}}^{\dagger}D_{di} - X_i{D_{fi}}^{\perp}D_{di}$$

$$\Psi_{25} = -{C_i}^T {{D_{fi}}^{\dagger}}^T {\beta_i}^T - {C_i}^T {{D_{fi}}^{\perp}}^T {S_i}^T {\beta_i}^T, \Psi_{45} = {D_{di}}^T {{D_{fi}}^{\dagger}}^T {\beta_i}^T + {D_{di}}^T {{D_{fi}}^{\perp}}^T {S_i}^T {\beta_i}^T$$

$$\Omega_{11} = A_j{}^T P_{11ij} + C_j^T D_{fi}^{\dagger}{}^T B_{fi}{}^T P_{12ij}{}^T - C_j^T D_{fi}^{\dagger}{}^T Y_i^T - \sigma P_{11ij} + P_{11ij} A_j$$

$$+ P_{12ij} B_{fi} D_{fi}{}^{\dagger} C_j - Y_i D_{fi}{}^{\perp} C_j$$

$$\Omega_{12} = A_j{}^T P_{12ij} + P_{12ij} A_i - \sigma P_{12ij} - P_{12ij} B_{fi} D_{fi}^{\dagger} C_i + Y_i D_{fi}{}^{\perp} C_i + C_j^T D_{fi}{}^{\dagger T} B_{fi}{}^T P_{22ij}$$

$$- C_j^T D_{fi}{}^{\perp T} Z_i{}^T$$

$$\Omega_{13} = P_{11ij} B_j + P_{12ij} B_i - P_{12ij} B_{fi} D_{fi}^{\dagger} D_i + P_{12ij} B_{fi} D_{fi}{}^{\dagger} D_j - Y_i D_{fi}{}^{\dagger} D_j + Y_i D_{fi}{}^{\perp} D_i$$

$$\Omega_{14} = P_{11ij} B_{dj} + P_{12ij} B_{fi} D_{fi}^{\dagger} D_{dj} - Y_i D_{fi}^{\perp} D_{dj}, \Omega_{15} = C_j{}^T D_{fi}{}^{\dagger T} \beta_i{}^T + C_j{}^T D_{fi}{}^{\perp T} S_i^T \beta_i^T$$

$$\Omega_{22} = A_i{}^T P_{22ij} - \sigma P_{22ij} - C_i{}^T D_{fi}^{\dagger}{}^T B_{fi}^T P_{22ij} + P_{22ij} A_i + C_i{}^T D_{fi}^{\perp}{}^T Z_i{}^T$$

$$- P_{22ij} B_{fi} D_{fi}{}^{\dagger} C_i + Z_i D_{fi}{}^{\perp} C_i$$

$$\Omega_{23} = P_{22ij}{}^T B_j + P_{22ij} B_i + P_{22ij} B_{fi} D_{fi}{}^{\dagger} D_j - Z_i D_{fi}{}^{\perp} D_j - P_{22ij} B_{fi} D_{fi}{}^{\dagger} D_i + Z_i D_{fi}{}^{\perp} D_i$$

$$\Omega_{24} = P_{12ij}{}^T B_{dj} + P_{22ij} B_{fi} D_{fi}{}^{\dagger} D_{dj} - Z_i D_{fi}{}^{\perp} D_{dj}, \Omega_{25} = -C_i{}^T D_{fi}{}^{\dagger T} \beta_i{}^T - C_i{}^T D_{fi}{}^{\perp T} S_i{}^T \beta_i{}^T$$

$$\Omega_{35} = D_j{}^T D_{fi}{}^{\dagger T} \beta_i{}^T + D_j{}^T D_{fi}{}^{\perp T} S_i{}^T \beta_i{}^T - D_i{}^T D_{fi}{}^{\dagger T} \beta_i{}^T - D_i{}^T D_{fi}{}^{\perp T} S_i{}^T \beta_i{}^T$$

$$\Omega_{45} = D_{dj}{}^T D_{fi}{}^{\dagger T} \beta_i{}^T + D_{dj}{}^T D_{fi}{}^{\perp T} S_i{}^T \beta_i{}^T$$

, $W_i = P_{12i} R_i, X_i = P_{22i} R_i, Y_i = P_{12ij} R_i, Z_i = P_{22ij} R_i$. *Moreover, parameters of fault detection filter are given by*

$$L_i = -B_{fi} D_{fi}{}^{\dagger} + R_i D_{fi}{}^{\perp}, H_i = \beta_i (D_{fi}{}^{\dagger} + S_i D_{fi}{}^{\perp})$$

where $R_i \in R^{n\times(m-q)}$ *and* $S_i \in R^{q\times(m-q)}$, *are additional variables that are introduced to provide more degree of freedom for* L_i *and* H_i.

Proof. As per A.4, in Chapter 2, we can say that there exist matrices $D_{fi}{}^{\dagger} \in R^{(q\times m)}$ and $D_{fi}{}^{\perp} \in R^{(m-q)\times m}$, such that the following condition is satisfied.

$$\begin{bmatrix} D_{fi}{}^{\dagger} \\ D_{fi}{}^{\perp} \end{bmatrix} D_{fi} = \begin{bmatrix} I_q \\ 0 \end{bmatrix}, rank(\begin{bmatrix} D_{fi}{}^{\dagger} \\ D_{fi}{}^{\perp} \end{bmatrix}) = m \tag{3.44}$$

Fault Isolation: We know that

$$F_i(s) G_{fi}(s) = (A_i + L_i C_i, B_{fi} + L_i D_{fi}, H_i C_i, H_i D_{fi}) \in RH_{\infty}^{q\times m}$$

and to achieve the fault isolation capability we require that

- $\|F_i G_{fi}\|_- \geq \beta_i \geq 1 \qquad \forall i \in \{1, 2, ...N\}$. For this,

- $F_iG_{fi} = \beta_i I$, $\forall i \in \{1, 2, ...N\}$, which can be obtained easily by setting
- $B_{fi} + L_iD_{fi} = 0$ and $H_iD_{fi} = \beta_i I_q$ in $F_i(s)G_{fi}(s)$. Next, from these two set equations, we can find

$$\begin{aligned} L_i &= -B_{fi}{D_{fi}}^{\dagger} + R_i{D_{fi}}^{\perp} \\ H_i &= \beta_i({D_{fi}}^{\dagger} + S_i{D_{fi}}^{\perp}) \end{aligned} \tag{3.45}$$

where$R_i \in R^{n\times(m-q)}$ and $S_i \in R^{q\times(m-q)}$, are additional variables that are to be found in H_∞ framework.

By aforementioned approach, with the setting of terms fault isolation framework is achieved. Now, for the desired L_i and H_i the only remaining part of the problem is to find out H_∞ norm of $F_iG_{di}, \forall i \in \{1, 2, ...N\}$. Using Lemma 3 (Appendix), during matched period,

$$\dot{V}_i(\tilde{x}(t)) \leq -\alpha_i V_i(\tilde{x}(t)) - r(t)^T r(t) + \gamma_i^2 \omega(t)^T \omega(t)\cdot \tag{3.46}$$

Considering the following Lyapunov function during matched time,

$$V_i(\tilde{x}(t)) = \tilde{x}^T(t)P_i\tilde{x}(t) \tag{3.47}$$

Differentiating (3.47), along the trajectory of (3.35)

$$\dot{V}_i(\tilde{x}(t)) = \dot{\tilde{x}}^T(t)P_i\tilde{x}(t) + \tilde{x}^T(t)P_i\dot{\tilde{x}}(t) \tag{3.48}$$

By substituting (3.47), (3.48), and $r(t)$ from (3.35) in (3.46),

$$\dot{\tilde{x}}^T(t)P_i\tilde{x}(t) + \tilde{x}^T(t)P_i\dot{\tilde{x}}(t) + \left[\tilde{C}_i\tilde{x}(t) + \tilde{D}_i\omega(t)\right]^T \left[\tilde{C}_i\tilde{x}(t) + \tilde{D}_i\omega(t)\right]$$

$$\leq -\alpha_i\tilde{x}^T(t)P_i\tilde{x}(t) + \gamma_i^2\omega(t)^T\omega(t) \tag{3.49}$$

Further, after substituting the expression for $\dot{\tilde{x}}(t)$ from (3.35) in (3.49),

$$\begin{aligned}&\left[\tilde{A}_i\tilde{x}(t)+\tilde{B}_i\omega(t)\right]^T P_i\tilde{x}(t)+\tilde{x}^T(t)P_i\left[\tilde{A}_i\tilde{x}(t)+\tilde{B}_i\omega(t)\right]\\&+(\tilde{x}^T(t)\tilde{C}_i^T+\omega^T(t)\tilde{D}_i^T)(\tilde{C}_i\tilde{x}(t)+\tilde{D}_i\omega(t))\\&\leq-\alpha_i\tilde{x}^T(t)P_i\tilde{x}(t)+\gamma_i^2\omega(t)^T\omega(t)\end{aligned}\tag{3.50}$$

(3.50) can be written easily in following form,

$$\begin{aligned}&\left[\tilde{x}^T(t)\tilde{A}_i^TP_i+\omega^T(t)\tilde{B}_i^TP_i\right]\tilde{x}(t)+\tilde{x}^T(t)P_i\tilde{A}_i\tilde{x}(t)\\&+\tilde{x}^T(t)P_i\tilde{B}_i\omega(t)+\tilde{x}^T(t)\tilde{C}_i^T\tilde{C}_i\tilde{x}(t)\\&+\tilde{x}^T(t)\tilde{C}_i^T\tilde{D}_i\omega(t)+\omega^T(t)\tilde{D}_i^T\tilde{C}_i\tilde{x}(t)\\&+\omega^T(t)\tilde{D}_i^T\tilde{D}_i\omega(t)+\alpha_i\tilde{x}^T(t)P_i\tilde{x}(t)-{\gamma_i}^2\omega^T(t)\omega(t)\leq 0\end{aligned}\tag{3.51}$$

Further, the above inequality can be written as

$$\begin{bmatrix}\tilde{x}^T(t) & \omega^T(t)\end{bmatrix}\begin{bmatrix}\tilde{A_i}^TP_i+P_i\tilde{A}_i+\tilde{C_i}^T\tilde{C}_i+\alpha_iP_i & P_i\tilde{B}_i+\tilde{C_i}^T\tilde{D}_i\\ * & \tilde{D_i}^T\tilde{D}_i-\gamma_i^2I\end{bmatrix}\begin{bmatrix}\tilde{x}(t)\\ \omega(t)\end{bmatrix}\leq 0 \tag{3.52}$$

For (3.52) to hold, it is required that

$$\begin{bmatrix}\tilde{A_i}^TP_i+P_i\tilde{A}_i+\tilde{C_i}^T\tilde{C}_i+\alpha_iP_i & P_i\tilde{B}_i+\tilde{C_i}^T\tilde{D}_i\\ * & \tilde{D_i}^T\tilde{D}_i-\gamma_i^2I\end{bmatrix}<0 \tag{3.53}$$

After Schur's compliment is applied to (3.53), we get

$$\begin{bmatrix}\tilde{A_i}^TP_i+P_i\tilde{A}_i+\alpha_iP_i & P_i\tilde{B}_i & \tilde{C_i}^T\\ * & -\gamma_i^2I & \tilde{D_i}^T\\ * & * & -I\end{bmatrix}<0 \tag{3.54}$$

Next, we substitute the expressions of L_i and H_i in (3.54), and get the following,

$$\begin{bmatrix} \Psi_{11} & \Psi_{12} & P_{11i}B_i + P_{12i}B_i & \Psi_{14} & \Psi_{15} \\ * & \Psi_{22} & {P_{12i}}^T B_i + P_{22i}B_i & \Psi_{24} & \Psi_{25} \\ * & * & -{\gamma_i}^2 I & 0 & 0 \\ * & * & * & -{\gamma_i}^2 I & \Psi_{45} \\ * & * & * & * & -I \end{bmatrix} < 0 \tag{3.55}$$

where,

$$\begin{aligned}
\Psi_{11} &= {A_i}^T P_{11i} + C_i^T {D_{fi}^{\perp}}^T {B_{fi}}^T {P_{12i}}^T + P_{11i}A_i - C_i^T {D_{fi}^{\perp}}^T {R_i}^T {P_{12i}}^T + P_{12i}B_{fi}{D_{fi}}^{\dagger}C_i \\
&\quad - P_{12i}R_i {D_{fi}}^{\perp} C_i + \alpha_i P_{11i} \\
\Psi_{12} &= {A_i}^T P_{12i} + P_{12i}A_i + \alpha_i P_{12i} - P_{12i}B_{fi}D_{fi}^{\dagger}C_i + {C_i}^T {D_{fi}^{\dagger}}^T {B_{fi}}^T P_{22i} - C_i {D_{fi}^{\dagger}}^T X_i^T \\
&\quad + P_{12i}R_i {D_{fi}}^{\perp} C_i \\
\Psi_{14} &= P_{11i}B_{di} + P_{12i}B_{fi}D_{fi}^{\dagger}D_{di} - P_{12i}R_i D_{fi}^{\perp} D_{di} \\
\Psi_{15} &= {C_i}^T {D_{fi}^{\dagger}}^T {\beta_i}^T + {C_i}^T {D_{fi}^{\perp}}^T {S_i}^T {\beta_i}^T \\
\Psi_{22} &= {A_i}^T P_{22i} + \alpha_i P_{22i} - {C_i}^T {D_{fi}^{\dagger}}^T B_{fi}^T P_{22i} + P_{22i}A_i + {C_i}^T {D_{fi}^{\perp}}^T {R_i}^T P_{22i} \\
&\quad - P_{22i}B_{fi}{D_{fi}}^{\dagger}C_i + P_{22i}R_i {D_{fi}}^{\perp} C_i \\
\Psi_{24} &= {P_{12i}}^T B_{di} + P_{22i}B_{fi}{D_{fi}}^{\dagger}D_{di} - P_{22i}R_i {D_{fi}}^{\perp} D_{di} \\
\Psi_{25} &= -{C_i}^T {D_{fi}^{\dagger}}^T {\beta_i}^T - {C_i}^T {D_{fi}^{\perp}}^T {S_i}^T {\beta_i}^T \\
\Psi_{45} &= {D_{di}}^T {D_{fi}^{\dagger}}^T {\beta_i}^T + {D_{di}}^T {D_{fi}^{\perp}}^T {S_i}^T {\beta_i}^T
\end{aligned}$$

The inequality (3.55) is a bilinear matrix inequality (BMI). In order to transform the BMI into a linear matrix inequality (LMI), we use the following substitutions; $P_{12i}R_i = W_i$ and $P_{22i}R_i = X_i$. Thus, LMI (3.42) for matched period is derived. Next we drive the conditions for unmatched period.

During unmatched period,

$$\dot{V}_{ij}(x(t)) \le \rho_i V_{ij}(x(t)) - y^T(t)y(t) + {\gamma_{ij}}^2 u^T(t)u(t) \tag{3.56}$$

Considering the following Lyapunov function during unmatched time,

$$V_{ij}(\tilde{x}(t)) = \tilde{x}^T(t)P_{ij}\tilde{x}(t)\cdot \tag{3.57}$$

Differentiating (3.57), along the trajectory of (3.36)

$$\dot{V}_{ij}(\tilde{x}(t)) = \dot{\tilde{x}}^T(t)P_{ij}\tilde{x}(t) + \tilde{x}^T(t)P_{ij}\dot{\tilde{x}}(t) \tag{3.58}$$

By substituting (3.57), (3.58), and $r(t)$ from (3.36) in (3.56),

$$\begin{aligned}&\dot{\tilde{x}}^T(t)P_{ij}\tilde{x}(t) + \tilde{x}^T(t)P_{ij}\dot{\tilde{x}}(t) + \left[\tilde{C}_{ij}\tilde{x}(t) + \tilde{D}_{ij}\omega(t)\right]^T\left[\tilde{C}_{ij}\tilde{x}(t) + \tilde{D}_{ij}\omega(t)\right]\\ &\leq \rho_i\ \tilde{x}^T(t)P_{ij}\tilde{x}(t) + \gamma_{ij}^2\omega(t)^T\omega(t)\end{aligned} \tag{3.59}$$

Further, after substituting the expression for $\dot{\tilde{x}}(t)$ from (3.36) in (3.59),

$$\begin{aligned}&\left[\tilde{A}_{ij}\tilde{x}(t) + \tilde{B}_{ij}\omega(t)\right]^T P_{ij}\tilde{x}(t) + \tilde{x}^T(t)P_{ij}\tilde{A}_{ij}\tilde{x}(t)\\ &+ (\tilde{x}^T(t)\tilde{C}_{ij}^T + \omega^T(t)\tilde{D}_{ij}^T)(\tilde{C}_{ij}\tilde{x}(t) + \tilde{D}_{ij}\omega(t))\\ &+ \tilde{x}^T(t)P_{ij}\tilde{B}_{ij}\omega(t) \leq \rho_i\ \tilde{x}^T(t)P_{ij}\tilde{x}(t) + \gamma_{ij}^2\omega(t)^T\omega(t)\end{aligned} \tag{3.60}$$

(3.60) can be written easily in following form,

$$\begin{aligned}&\left[\tilde{x}^T(t)\tilde{A}_{ij}^TP_i + \omega^T(t)\tilde{B}_{ij}^TP_i\right]\tilde{x}(t) + \tilde{x}^T(t)P_{ij}\tilde{A}_{ij}\tilde{x}(t)\\ &+ \tilde{x}^T(t)P_{ij}\tilde{B}_{ij}\omega(t) + \tilde{x}^T(t)\tilde{C}_{ij}^T\tilde{C}_{ij}\tilde{x}(t)\\ &+ \tilde{x}^T(t)\tilde{C}_{ij}^T\tilde{D}_{ij}\omega(t) + \omega^T(t)\tilde{D}_{ij}^T\tilde{C}_{ij}\tilde{x}(t)\\ &+ \omega^T(t)\tilde{D}_{ij}^T\tilde{D}_{ij}\omega(t) - \rho_i\ \tilde{x}^T(t)P_{ij}\tilde{x}(t)\\ &- {\gamma_{ij}}^2\omega^T(t)\omega(t) \leq 0\end{aligned} \tag{3.61}$$

Further, the inequality (3.61) can be written as

$$\begin{bmatrix}\tilde{x}^T(t) & \omega^T(t)\end{bmatrix} M \begin{bmatrix}\tilde{x}(t)\\ \omega(t)\end{bmatrix} \leq 0 \tag{3.62}$$

where, M is

$$\begin{bmatrix} \tilde{A_{ij}}^T P_{ij} + P_{ij}\tilde{A}_{ij} + \tilde{C_{ij}}^T \tilde{C}_{ij} - \rho_i P_{ij} & P_{ij}\tilde{B}_{ij} + \tilde{C_{ij}}^T \tilde{D}_{ij} \\ * & \tilde{D_{ij}}^T \tilde{D}_{ij} - \gamma_{ij}^2 I \end{bmatrix}$$

To hold (3.62), it is required that

$$\begin{bmatrix} \tilde{A_{ij}}^T P_{ij} + P_{ij}\tilde{A}_{ij} + \tilde{C_{ij}}^T \tilde{C}_{ij} - \rho_i P_{ij} & P_{ij}\tilde{B}_{ij} + \tilde{C_{ij}}^T \tilde{D}_{ij} \\ * & \tilde{D_{ij}}^T \tilde{D}_{ij} - \gamma_{ij}^2 I \end{bmatrix} < 0 \tag{3.63}$$

After Schuar's compliment is applied to (3.63), we get

$$\begin{bmatrix} \tilde{A_{ij}}^T P_{ij} + P_{ij}\tilde{A}_{ij} - \rho_i P_{ij} & P_{ij}\tilde{B}_{ij} & \tilde{C_{ij}}^T \\ * & -\gamma_{ij}^2 I & \tilde{D_{ij}}^T \\ * & * & -I \end{bmatrix} < 0 \tag{3.64}$$

Next, we substitute the expressions of L_i and H_i from (3.45) in (3.64), and get the following inequality

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & \Omega_{15} \\ * & \Omega_{22} & \Omega_{23} & \Omega_{24} & \Omega_{25} \\ * & * & -\gamma_{ij}{}^2 I & 0 & \Omega_{35} \\ * & * & * & -\gamma_{ij}{}^2 I & \Omega_{45} \\ * & * & * & * & -I \end{bmatrix} < 0 \tag{3.65}$$

where,

$$\begin{aligned}
\Omega_{11} &= A_j{}^T P_{11ij} + C_j^T D_{fi}{}^{\dagger T} B_{fi}{}^T P_{12ij}{}^T + P_{11ij} A_j - C_j^T D_{fi}{}^{\perp T} R_i{}^T P_{12ij}{}^T \\
&\quad + P_{12ij} B_{fi} D_{fi}{}^{\dagger} C_j - P_{12i} R_i D_{fi}{}^{\perp} C_i - \rho_i P_{11ij} \\
\Omega_{12} &= A_j{}^T P_{12ij} + P_{12ij} A_i - P_{12i} B_{fi} D_{fi}^{\dagger} C_i + C_j{}^T D_{fi}{}^{\dagger T} B_{fi}{}^T P_{22ij} \\
&\quad - C_j D_{fi}{}^{\dagger T} R_i{}^T P_{22ij} + P_{12i} R_i D_{fi}{}^{\perp} C_i - \rho_i P_{12ij} \\
\Omega_{13} &= P_{11ij} B_j + P_{12ij} B_i - P_{12ij} B_{fi} D_{fi}^{\dagger} D_i + P_{12ij} B_{fi} D_{fi}{}^{\dagger} D_j - P_{12ij} R_i D_{fi}{}^{\dagger} D_j \\
&\quad + P_{12ij} R_i D_{fi}{}^{\perp} D_i \\
\Omega_{14} &= P_{12ij} B_{fi} D_{fi}^{\dagger} D_{dj} - P_{12ij} R_i D_{fi}^{\perp} D_{dj} + P_{11ij} B_{dj}
\end{aligned}$$

$$\Omega_{15} = {C_j}^T {D_{fi}}^{\dagger^T} {\beta_i}^T + {C_j}^T {D_{fi}}^{\perp^T} {S_i}^T {\beta_i}^T$$

$$\Omega_{22} = {A_i}^T P_{22ij} - \rho_i P_{22ij} - {C_i}^T {D_{fi}^{\dagger}}^T B_{fi}^T P_{22ij} + P_{22ij} A_i + {C_i}^T {D_{fi}^{\perp}}^T {R_i}^T P_{22ij}$$
$$- P_{22ij} B_{fi} {D_{fi}}^{\dagger} C_i + P_{22ij} R_i {D_{fi}}^{\perp} C_i$$

$$\Omega_{23} = {P_{12ij}}^T B_j + P_{22ij} B_i - P_{22ij} B_{fi} {D_{fi}}^{\dagger} D_i + P_{22ij} R_i {D_{fi}}^{\perp} D_i + P_{22ij} B_{fi} {D_{fi}}^{\dagger} D_j$$
$$- P_{22ij} R_i {D_{fi}}^{\perp} D_j$$

$$\Omega_{24} = {P_{12ij}}^T B_{dj} + P_{22ij} B_{fi} {D_{fi}}^{\dagger} D_{dj} - P_{22ij} R_i {D_{fi}}^{\perp} D_{dj}$$

$$\Omega_{25} = -{C_i}^T {D_{fi}}^{\dagger^T} {\beta_i}^T - {C_i}^T {D_{fi}}^{\perp^T} {S_i}^T {\beta_i}^T$$

$$\Omega_{35} = {D_j}^T {D_{fi}}^{\dagger^T} {\beta_i}^T + {D_j}^T {D_{fi}}^{\perp^T} {S_i}^T {\beta_i}^T - {D_i}^T {D_{fi}}^{\dagger^T} {\beta_i}^T - {D_i}^T {D_{fi}}^{\perp^T} {S_i}^T {\beta_i}^T$$

$$\Omega_{45} = {D_{dj}}^T {D_{fi}}^{\dagger^T} {\beta_i}^T + {D_{dj}}^T {D_{fi}}^{\perp^T} {S_i}^T {\beta_i}^T$$

The inequality (3.65) is a bilinear matrix inequality (BMI). In order to transform the BMI into a Linear Matrix Inequality (LMI), we use the following substitutions; $P_{12ij} R_i = Y_i, P_{22ij} R_i = Z_i$. Thus, LMI (3.43) is derived. In order to satisfy LMIs (3.42) and (3.43), L_i and H_i must be computed according to (3.45). Now, the proof is completed which ensures the fault sensitivity level to be $\beta_i, \forall i \in \{1, 2, ...N\}$, disturbance attenuation level $\gamma_i, \forall i \in \{1, 2, ...N\}$ and fault isolation capability achieved by the derived filter. □

Remark 1. *In the fault detection (FD) framework, the residual signal should be generated such that it contains the information of faults only. Due to this reason, effect of all other signals, that is, $(u(t), d(t))$ on residual signal needs to be minimized / eliminated. To achieve this purpose, $u(t)$ is included with $d(t)$ in $\omega(t)$ vector. However, for control purpose, $u(t)$ can be dealt separately. Moreover, control inputs $u(t)$, in general, are bounded in control loops. The bounds of $u(t)$ depends on particular application. Here, the only reason to assume norm bounded $u(t)$ is to study the effect of control input $(u(t))$ and disturbance $(d(t))$ on residual signal $(r(t))$ in unified way.*

In the next subsection, we present the algorithm in stepwise simplified form to design our objective filter, which is based on the results derived in Theorem 2.

3.3.4 Algorithm

Let the model of the switched system is given as in 3.33. By taking into account the assumptions, which were made earlier.

1. Check the detectability of all subsystems (modes), i.e., whether (A_i, C_i) is detectable $\forall i \in \{1, 2, ...N\}$, if yes then proceed to next step, if no, then it is not possible to proceed

2. Find $D_{fi}^{\dagger}$ and $D_{fi}^{\perp}$ such that
$\begin{bmatrix} {D_{fi}}^{\dagger} \\ {D_{fi}}^{\perp} \end{bmatrix} D_{fi} = \begin{bmatrix} I_{nf} \\ 0 \end{bmatrix}$, rank $\left(\begin{bmatrix} {D_{fi}}^{\dagger} \\ {D_{fi}}^{\perp} \end{bmatrix} \right) = m.$

3. Set the ADT parameters, $\mu_1, \mu_2, \alpha_i, \rho_i$, and $\beta_i = 1\ \forall \quad i \in \{1, 2, ...N\}$ then solve the LMIs (3.39)-(3.43) simultaneously to get the optimal values of γ_i, $\forall \quad i \in \{1, 2, ...N\}$.

4. Find variables R_i and $S_i\ \forall \quad i \in \{1, 2, ...N\}$ from step 3) and set the filter parameters as formulated below,
$L_i = -B_{fi} {D_{fi}}^{\dagger} + R_i {D_{fi}}^{\perp}, H_i = \beta_i({D_{fi}}^{\dagger} + S_i D_{fi}^{\perp})$.

3.3.5 Threshold Computation and Residual Evaluation

Residual evaluation and threshold computation are employed as discussed in the preceding Section 3.3, as follows.

$$J_{RMS} = \parallel r(t) \parallel_{RMS} = \left(\frac{1}{T}\int_{t}^{t+T} \parallel r(\tau) \parallel^2 d\tau\right)^{\frac{1}{2}}$$

where, T is the evaluation window.

Threshold computation is defined as

$$J_{th,RMS,2} = \sup_{\|\omega(t)\|_2 \leq \delta_{d,2} + \delta_{u,2}, f(t)=0} J_{RMS}, \tag{3.66}$$

and computed as

$$J_{th,RMS,2} = \frac{\gamma_i^*(\delta_{d,2} + \delta_{u,2})}{\sqrt{T}} \tag{3.67}$$

where $\gamma_i^* = min(\gamma_i)$, $\delta_{d,2}$ denotes the set of L_2-norm bounded disturbance signals and $\delta_{u,2}$ denotes the set of L_2-norm bounded input signals.

3.3.6 Application to the Case Studies: HiMAT Vehicle and Buck-boost Converter

In this section, proposed framework is utilized for fault detection and isolation in case studies of Highly Maneuverable Aircraft Technology (HiMAT) and Buck-boost converter. The Simulation platform for both cases is the same for time 30s. Subsystem 1 is activated when $\sigma(t) = 1$ and subsystem 2 when $\sigma(t) = 0$. In case of HiMAT, one fault $f(t) = 0.5$ is applied while in case of Buck-boost converter, two faults, $f_1(t) = 1$ and $f_2(t) = 0.5$ are simulated. Details of switching behaviour of corresponding subsystems, filters and faults can be found in Fig.3.9 and Fig.3.14, respectively. In addition, for both studies we choose, $\beta_i = 1 \forall \quad i \in \{1, 2\}$, (fault sensitivity levels) and $\mu_1 = 1.5, \mu_2 = 1.5$ and $\alpha_i = 0.5, \rho_i = 0.5$, parameters of ADT. To this end, switching signal $\sigma(t)$ is applied with ADT value of 1.6218, that is, switching interval between any two subsystems is greater than 1.6218. Further, for simulation study we take the disturbance signal as L_2 norm bounded with $\delta_{d,2} \leq 1$ for each mode. In practice noise signal is of stochastic nature, for simplification, here we assume the noise signal of deterministic nature for each mode.

Case Study: HiMAT Vehicle

First, we consider the model of a HiMAT (Highly Maneuverable Aircraft Technology) ([98, 99]). Which can be considered as a two dimensional switched system, where $x_1(t)$ and $x_2(t)$ are the angle of attack and pitch rate; respectively. The dynamics of two modes of the switched systems are given below.

$$\left[\begin{array}{c|c|c|c} A_1 & B_1 & B_{d1} & B_{f1} \\ \hline C_1 & D_1 & D_{d1} & D_{f1} \end{array}\right]$$

$$= \left[\begin{array}{cc|c|c|c} -1.35 & -0.98 & 1.7 & -0.1 & 1.7 \\ 17.1 & -1.85 & 0.9 & -0.09 & 0.9 \\ \hline 1 & 0 & 1.10 & 0.3 & 2.1 \\ 0 & 1 & 0.10 & 0 & 0.1 \end{array}\right]$$

$$\left[\begin{array}{c|c|c|c} A_2 & B_2 & B_{d2} & B_{f2} \\ \hline C_2 & D_2 & D_{d2} & D_{f2} \end{array}\right]$$

$$= \left[\begin{array}{cc|c|c|c} -1.87 & -0.98 & 1.9 & 0.2 & 1.9 \\ 12.6 & -2.63 & 3.8 & 0.1 & 3.8 \\ \hline 1 & 0 & 1.9 & 0.4 & 1.9 \\ 0 & 1 & 2.30 & 0.9 & 2.3 \end{array}\right]$$

Results and Discussion

By using Theorem 2, following parameters of filter are obtained

$$L_1 = \begin{bmatrix} -0.8417 & 0.6766 \\ -0.4341 & 0.1159 \end{bmatrix}, H_1 = \begin{bmatrix} 0.4568 & 0.4065 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} -0.4050 & -0.5047 \\ -0.8102 & -1.0042 \end{bmatrix}, H_2 = \begin{bmatrix} 1.1359 & -0.5035 \end{bmatrix}$$

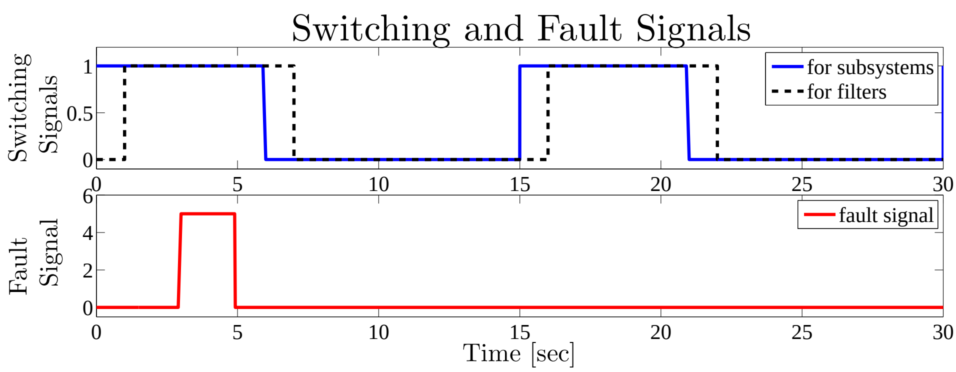

Figure 3.9: Switching signals for subsystems and Fault signal

We find disturbance attenuation levels $\gamma_1 = 0.1473$, and $\gamma_2 = 0.1367$ from LMIs solution. Moreover, the equations $B_{fi} + L_i D_i = 0$ and $H_i D_i = \beta_i I_q$ are also satisfied. Then, fault is simulated as in Fig.3.9.

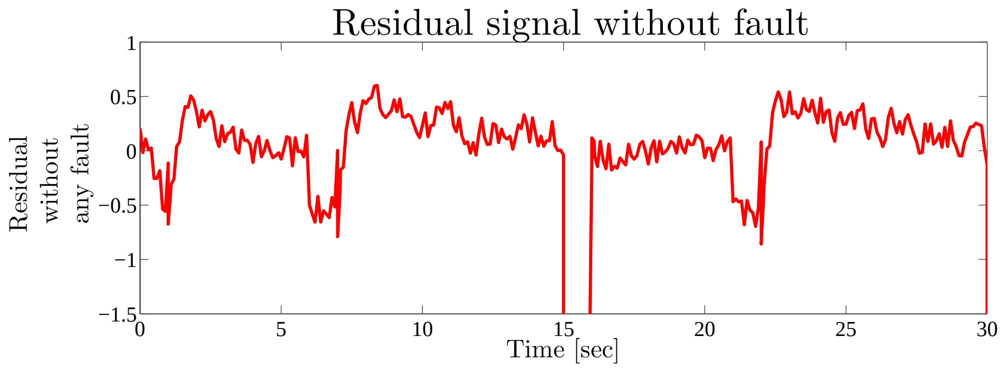

Figure 3.10: Residual signal without fault

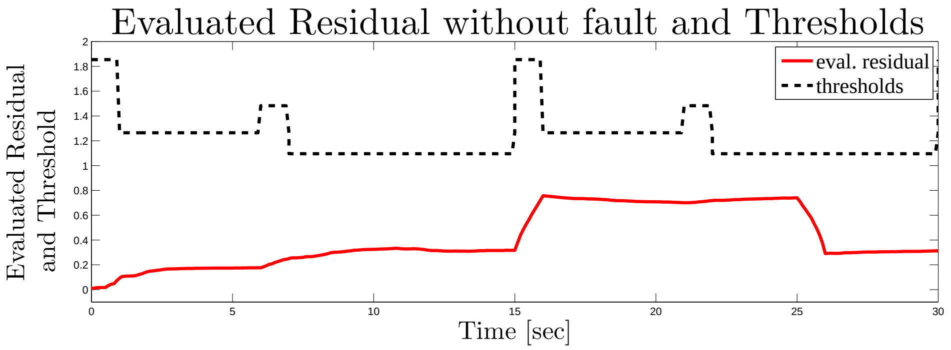

Figure 3.11: Evaluated Residual signal without fault

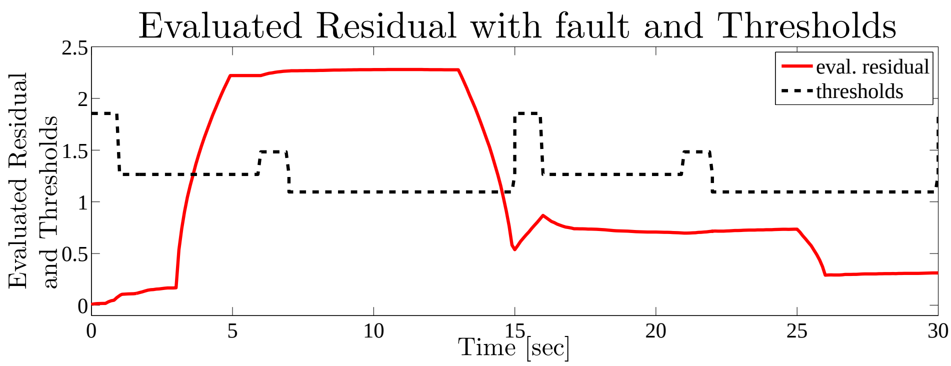

Figure 3.12: Evaluated residual signal with fault and threshold

Under the above mentioned setting, The residual signals for the system in absence of faults is depicted in Fig. 3.10. It can be seen that the residual signal is non-zero even in case of no fault. To this end, proper fault detection is not possible, and false alarms may be generated. Due to this reason, residual signals need to be evaluated and then a threshold level has to be set for detecting the faults. Using (3.67), threshold value is set to be 0.2072. Evaluated residuals and thresholds have been plotted in figures Fig. (3.11) in case of no fault and in Fig.(3.12) for faulty case, whereas window size of T=10 is used for evaluating the residual signal. It is easy to see that the fault is detected effectively in very short span of time, when evaluated residual signal crosses the threshold level.

Remark 2. *In the discussed case of HiMAT system, we introduced only one fault, according to our assumption number of faults should be less than outputs. Therefore, fault isolation is not applicable in this case. To demonstrate the fault isolation along with fault detection successfully, we consider next the example of Buck-boost converter with the case of two faults occurrence.*

Case Study: Buck-boost Converter

In this subsection, we consider the Buck-boost converter switched system ([3, 100]) for simulating the proposed design technique, the circuit is depicted in Fig.3.13. and the

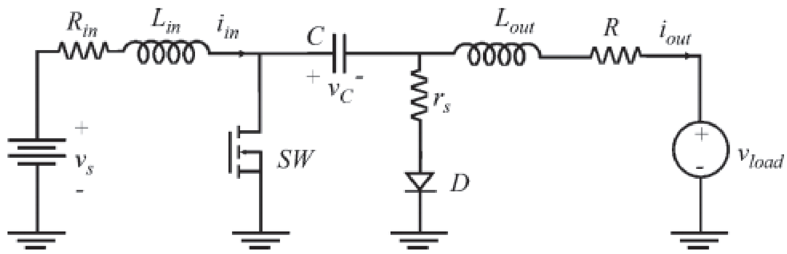

Figure 3.13: Buck-boost converter circuit diagram [3]

dynamic model of this system in two switching modes is given below:

$$A_1 = \begin{bmatrix} -\frac{R_{in}+r_s}{L_{in}} & \frac{r_s}{L_{in}} & -\frac{1}{L_{in}} \\ \frac{r_s}{L_{out}} & -\frac{R+r_s}{L_{out}} & 0 \\ \frac{1}{C} & 0 & 0 \end{bmatrix}, B_1 = \begin{bmatrix} \frac{1}{L_{in}} \\ 0 \\ 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -\frac{R_{in}}{L_{in}} & 0 & 0 \\ 0 & -\frac{R}{L_{out}} & -\frac{1}{L_{out}} \\ 0 & \frac{1}{C} & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ -\frac{1}{L_{out}} \\ 0 \end{bmatrix}$$

where $x = [i_{in}\ i_{out}\ v_c]$ is the state vector of the converter, and we assume that measurements of all state variables are available. The parameter values of Buck-boost converter are $R_{in} = 30$ ohms, $L_{in} = 20$ mH, C = 20 μ F, $r_s = 10$ ohms, $L_{out} = 20$ mH, R = 30 ohms, and $v_s = 15$ volts. According to these values, system matrices are,

$$A_1 = \begin{bmatrix} -2000 & 500 & -50 \\ 500 & -2000 & 0 \\ 50000 & 0 & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 50 \\ 0 \\ 0 \end{bmatrix}, C_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, D_1 = \begin{bmatrix} 1.5 \\ 0 \\ 0 \end{bmatrix},$$

$$B_{d1} = \begin{bmatrix} -0.1 & 0.03 \\ -0.2 & 0.1 \\ -0.01 & 0.1 \end{bmatrix}, D_{d1} = \begin{bmatrix} 0.02 & -0.1 \\ 0.01 & -0.2 \\ -0.01 & 0.02 \end{bmatrix}, B_{f1} = \begin{bmatrix} 50 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -1500 & 0 & 0 \\ 0 & -1500 & -50 \\ 0 & 50000 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ -50 \\ 0 \end{bmatrix}, C_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, D_2 = \begin{bmatrix} 1.5 \\ 0 \\ 0 \end{bmatrix},$$

$$B_{d2} = \begin{bmatrix} -0.01 & -0.03 \\ 0.1 & -0.16 \\ -0.2 & 0.1 \end{bmatrix}, D_{d2} = \begin{bmatrix} 0.11 & 0.3 \\ 0.2 & -0.01 \\ -0.1 & -0.05 \end{bmatrix}, B_{f2} = \begin{bmatrix} 0 & 0 \\ -50 & 0 \\ 0 & 0 \end{bmatrix},$$

$$D_{f1} = \begin{bmatrix} 1.5 & 1.10 \\ 1.70 & 1.90 \\ 0 & 0 \end{bmatrix}, D_{f2} = \begin{bmatrix} 1.5 & 0 \\ 0 & 1.9 \\ 0 & 0 \end{bmatrix}$$

Results and Discussion

By solving LMIs (3.39-3.43)of the proposed scheme, following design parameters are obtained.

$$L_1 = \begin{bmatrix} -0.2041 & -0.4082 & 0 \\ 1.7347 & -1.5306 & 0 \\ 0 & 0 & 0.0938 \end{bmatrix}, H_1 = \begin{bmatrix} 1.9388 & -1.7347 \\ -1.1224 & 1.5306 \\ -1.0575 & 0.9973 \end{bmatrix}^T,$$

$$L_2 = \begin{bmatrix} -0.9178 & 1.1096 & 0 \\ 0.5479 & -2.6027 & 0 \\ 0 & 0 & 0.1143 \end{bmatrix}, H_2 = \begin{bmatrix} 0.9589 & -0.5479 \\ -2.0548 & 2.6027 \\ 3.6766 & -4.4402 \end{bmatrix}^T.$$

Then, we simulate the systems as in Fig.3.14. The residual signals for the system in absence and presence of faults are depicted in figures, Fig. (3.15) and Fig. (3.16). Here it is easy to see that $f_1(t)$ effects only the residual 1 whereas $f_2(t)$ effect exclusively residual 2. In this way, not only both faults are successfully detected but also isolated (located). It is worth noticing that two design requirements for fault isolation, $B_{fi} + L_i D_i = 0$ and $H_i D_i = \beta_i I_q$ are also satisfied. Finally, to avoid false alarms, residuals are evaluated by RMS and thresholds are set according to 3.66 and 3.67, respectively, which are depicted in Fig.3.17 and Fig.3.18. For residual evaluation window size T is set to 10 while Thresholds for residual signal 1 and residual signal 2 are 0.0390 and 0.0410 respectively in this case. Fault is detected properly when residual evaluated signals 1 and residual evaluated signal 2 cross their respective thresholds values.

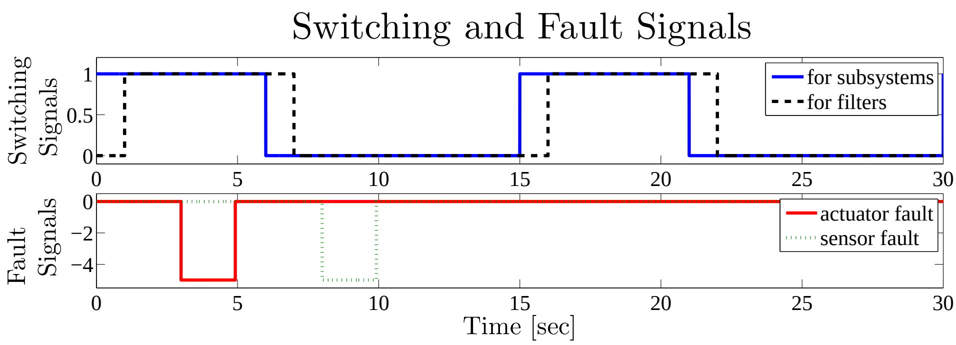

Figure 3.14: Switching signal $\sigma(t)$ for subsystems and Fault signals

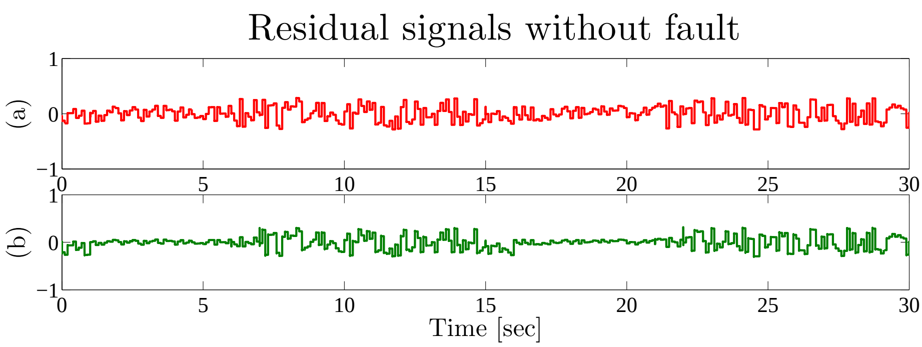

Figure 3.15: (a): Residual signal 1 without any fault (b): Residual signal 2 without any fault

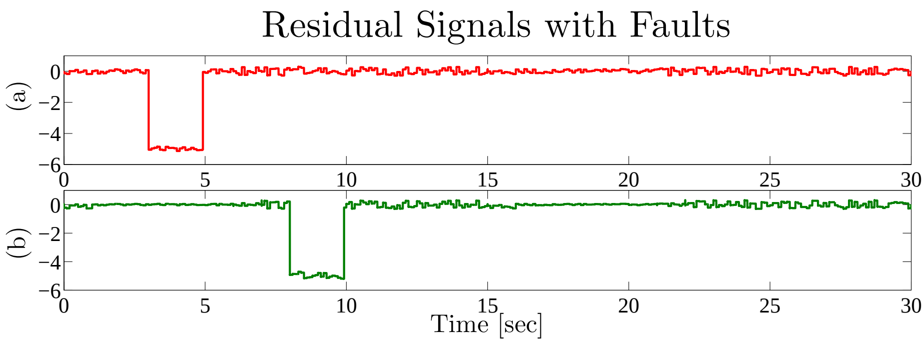

Figure 3.16: (a): Residual signal 1 with fault $f_1(t)$(b): Residual signal 2 with fault $f_2(t)$

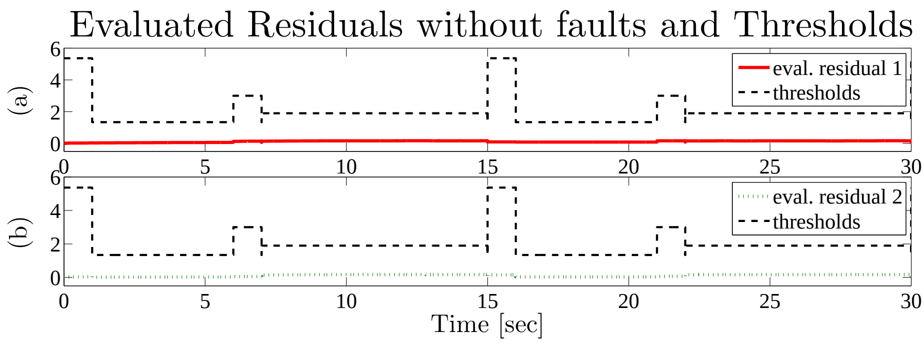

Figure 3.17: (a) and (b): Evaluated residual signals without fault and thresholds

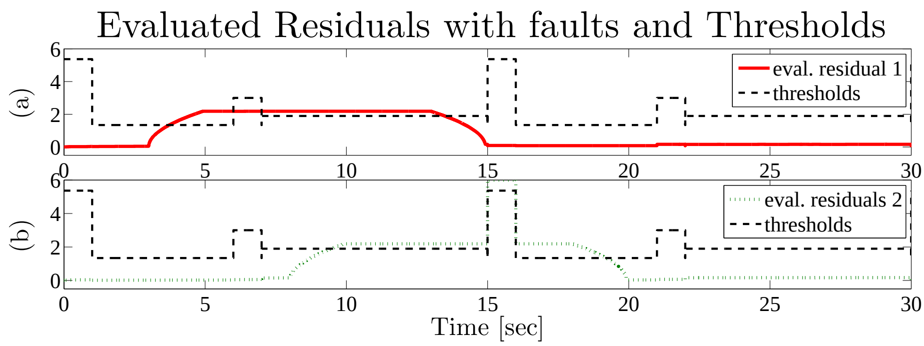

Figure 3.18: Evaluated residual signals with fault and thresholds

Concluding Remarks

In this section, the problem of fault detection and isolation for continuous-time switched control systems under asynchronous switching has been investigated and solution is provided in the form of a mixed H_-/H_∞ filtering approach using model-based FD Framework. Average dwell time constraint has been considered while employing piecewise

Lyapunov function. The results are derived in the form of LMIs. Advantage of the proposed solution is that not only fault detection but also fault isolation is possible, which has been demonstrated by design of case studies, HiMAT and Buck-boost power converter.

In the sequel, the same problem of fault detection and isolation is studied for uncertain switched systems. In the considered uncertain switched system, uncertainty is assumed to be of norm-bounded type. The problem is also considered by computing adaptive threshold for efficient FDI results.

3.4 Fault Detection and Isolation in Uncertain Switched Systems

The problem of fault detection and isolation for uncertain continuous-time linear switched systems in the presence of disturbances and noise is addressed in this section. A robust residual generator is proposed which is based on asynchronously switching filters. Considered system is assumed to be uncertain in norm bounded sense. Model uncertainties make further difficult the fault detection and isolation (FDI) problem along with unknown inputs when using model-based FDI Approach. To address the issue, FDI problem is formulated as mixed H_-/H_∞ filtering problem. In proposed H_-/H_∞ technique, the effect of fault on residual signal while influence of unknown inputs (disturbances and noise) on residual is optimized. In addition proposed filter has prominence of having fault isolation capability along with fault detection. To improve the fault detection capability adaptive threshold is designed which takes into account local disturbance levels, the current operational mode and applied input signal. To deal with the major issue of asynchronous switching, during matched and unmatched time of switched systems, a piecewise Lyapunov function along with average dwell time scheme is employed, and the results are derived in terms of linear matrix inequalities. Then, FDI scheme is designed for boost converter switched system application. Finally designed filter parameters are simulated to illustrate the efficacy of the proposed framework.

In this Section, we present a solution to the fault detection and isolation problem for uncertain continuous-time switched systems under asynchronous switching case. This work is extension of our results [101], by taking norm-bounded model uncertainties into account and instead of fixed threshold, which is conservative, proposing the adaptive threshold setting for this problem. Our proposed solution has the following prominent features. First, the attention is given to the optimal solution in the sense that the effect of unknown input on residual signal is minimized whereas that of the fault is maximized simultaneously using H_-/H_∞ optimization index. Second, a contribution of the work is to achieve the fault isolation capability in straightforward way along with fault detection while investigating the complex asynchronous problem of switched systems. In this research, the average dwell time (ADT) constraint related to switching phenomenon is considered.

The rest of the section is organized as follows: In Subsection 3.4.1 problem is formulated and in Subsection 3.4.2 solution is proposed. In Subsection 3.4.3, adaptive threshold is computed for the case study and residual signals are evaluated. In Subsection 3.4.4, application results are demonstrated to show the effectiveness of the approach.

3.4.1 Problem Formulation: Fault Detection and Isolation in Uncertain Switched Systems

Systems Description

Following class of continuous-time switched systems, is considered

$$\begin{aligned} \dot{x}(t) &= \bar{A}_{\sigma(t)}x(t) + \bar{B}_{\sigma(t)}u(t) + \bar{B}_{d\sigma(t)}d(t) + B_{f\sigma(t)}f(t) \\ y(t) &= \bar{C}_{\sigma(t)}x(t) + \bar{D}_{\sigma(t)}u(t) + \bar{D}_{d\sigma(t)}d(t) + D_{f\sigma(t)}f(t) \end{aligned} \tag{3.68}$$

where $x(t) \in \Re^n$ is the state vector, $u(t) \in \Re^r$ is the control input vector, $y(t) \in \Re^m$ is the output vector, $d(t) \in \Re^p$ is the unknown inputs (disturbances, noise) vector, $f(t) \in \Re^q$ is the vector, $\sigma(t)$ is a switching signal which is piecewise constant function $\sigma : [0, \infty) \longrightarrow \rho$. Such a function σ has a finite number of switching times and takes a constant value on every interval between two consecutive switching times. The role of $\sigma(t)$ is to specify, at each instant t, the index $\sigma(t) \in \rho$ of the active subsystem.

$\bar{A}_{\sigma(t)}, \bar{B}_{\sigma(t)}, \bar{C}_{\sigma(t)}, \bar{D}_{\sigma(t)}, \bar{B}_{d\sigma(t)}, \bar{D}_{d\sigma(t)}, B_{f\sigma(t)}, D_{f\sigma(t)}$ are the systems, disturbances and fault coupling matrices with appropriate dimensions and

$$\begin{aligned}
&\bar{A}_{\sigma(t)} = A_{\sigma(t)} + \Delta A_{\sigma(t)}, \bar{B}_{\sigma(t)} = B_{\sigma(t)} + \Delta B_{\sigma(t)},\\
&\bar{C}_{\sigma(t)} = C_{\sigma(t)} + \Delta C_{\sigma(t)}, \bar{D}_{\sigma(t)} = D_{\sigma(t)} + \Delta D_{\sigma(t)},\\
&\bar{B}_{d\sigma(t)} = B_{d\sigma(t)} + \Delta B_{d\sigma(t)}, \bar{D}_{d\sigma(t)} = D_{d\sigma(t)} + \Delta D_{d\sigma(t)}
\end{aligned}$$

where,

$$\Delta A_{\sigma(t)}, \Delta B_{\sigma(t)}, \Delta C_{\sigma(t)}, \Delta D_{\sigma(t)}, \Delta B_{d\sigma(t)}, \Delta D_{d\sigma(t)}$$

are norm bounded uncertainties with following definition

$$\begin{aligned}
&\Delta A_{\sigma(t)} = E_{\sigma(t)}\Delta(t)G_{\sigma(t)}, \Delta B_{\sigma(t)} = E_{\sigma(t)}\Delta(t)H_{\sigma(t)}\\
&\Delta B_{d\sigma(t)} = E_{\sigma(t)}\Delta(t)J_{\sigma(t)}, \Delta C_{\sigma(t)} = F_{\sigma(t)}\Delta(t)G_{\sigma(t)}\\
&\Delta D_{\sigma(t)} = F_{\sigma(t)}\Delta(t)H_{\sigma(t)}, \Delta D_{d\sigma(t)} = F_{\sigma(t)}\Delta(t)J_{\sigma(t)}
\end{aligned}$$

where $E_{\sigma(t)}, F_{\sigma(t)}, G_{\sigma(t)}, H_{\sigma(t)}, J_{\sigma(t)}$ are known matrices of appropriate dimensions and $\Delta(t)$ is unknown but norm bounded $\Delta(t)^T\Delta(t) \leq I$

We denote the association of these matrices with particular switching signal instant $\sigma(t) = i$ by $A_{\sigma(t)} = A_i$, where $i = 1, 2, ...N$, number of subsystems involved.

In order to generate residual signal fault detection filter, 2.9, is used as residual generator, which is as follows.

$$\begin{aligned}
&\dot{\hat{x}}(t) = A_{\sigma'(t)}\hat{x}(t) + B_{\sigma'(t)}u(t) - L_{\sigma'(t)}(y(t) - \hat{y}(t))\\
&\hat{y}(t) = C_{\sigma'(t)}\hat{x}(t) + D_{\sigma'(t)}u(t)\\
&r(t) = H_{\sigma'(t)}(y(t) - \hat{y}(t))
\end{aligned} \tag{3.69}$$

During Matched Period:

During the matched period, ith subsystem and ith filter are in operation, refer to Fig.2.7. We augment the switched system (3.68) and detection filter (3.69) into the following compact representation during matched period

$$\dot{\tilde{x}}(t) = (\tilde{A}_i + \Delta\tilde{A}_i)\tilde{x}(t) + \breve{B}_{fi}f(t) + (\tilde{B}_i + \Delta\tilde{B}_i))\omega(t)$$
$$r(t) = (\tilde{C}_i + \Delta\tilde{C}_i)\tilde{x}(t) + \breve{D}_{fi}f(t) + (\tilde{D}_i + \Delta\tilde{D}_i)\omega(t) \quad (3.70)$$

where,

$$\tilde{x}(t) = \begin{bmatrix} x(t)^T & \hat{x}(t)^T \end{bmatrix}^T, \omega(t) = \begin{bmatrix} u(t)^T & d(t)^T \end{bmatrix}^T$$

and

$$\tilde{A}_i = \begin{bmatrix} A_i & 0 \\ -L_iC_i & A_i + L_iC_i \end{bmatrix}, \Delta\tilde{A}_i = \begin{bmatrix} \Delta A_i & 0 \\ -L_i\Delta C_i & 0 \end{bmatrix}$$
$$\tilde{B}_i = \begin{bmatrix} B_i & B_{di} \\ B_i & -L_iD_{di} \end{bmatrix} \Delta\tilde{B}_i = \begin{bmatrix} \Delta B_i & \Delta B_{di} \\ -L_i\Delta D_i & -L_i\Delta D_{di} \end{bmatrix}$$
$$\tilde{C}_i = \begin{bmatrix} H_iC_i & -H_iC_i \end{bmatrix}, \Delta\tilde{C}_i = \begin{bmatrix} H_i\Delta C_i & 0 \end{bmatrix}$$
$$\tilde{D}_i = \begin{bmatrix} 0 & H_iD_{di} \end{bmatrix}, \Delta\tilde{D}_i = \begin{bmatrix} H_i\Delta D_i & H_i\Delta D_{di} \end{bmatrix}$$
$$\breve{B}_{fi} = \begin{bmatrix} B_{fi} \\ 0 \end{bmatrix}, \breve{D}_{fi} = \begin{bmatrix} H_iD_{fi} \end{bmatrix}$$

During Unmatched Period: During the unmatched period, jth subsystem and ith filter are switched together, refer to Fig.2.7. We augment the switched system (3.68) and detection filter (3.69) into the following compact representation during unmatched period

$$\dot{\tilde{x}}(t) = (\tilde{A}_{ij} + \Delta\tilde{A}_{ij})\tilde{x}(t) + \breve{B}_{fij}f(t) + (\tilde{B}_{ij} + \Delta\tilde{B}_{ij}))\omega(t)$$
$$r(t) = (\tilde{C}_{ij} + \Delta\tilde{C}_{ij})\tilde{x}(t) + \breve{D}_{fij}f(t) + (\tilde{D}_{ij} + \Delta\tilde{D}_{ij})\omega(t) \quad (3.71)$$

where,

$$\tilde{x}(t) = \begin{bmatrix} x(t)^T & \hat{x}(t)^T \end{bmatrix}^T, \omega(t) = \begin{bmatrix} u(t)^T & d(t)^T \end{bmatrix}^T$$

and

$$\tilde{A_{ij}} = \begin{bmatrix} A_j & 0 \\ -L_iC_j & A_i + L_iC_i \end{bmatrix}, \Delta\tilde{A_{ij}} = \begin{bmatrix} \Delta A_j & 0 \\ -L_i\Delta C_j & 0 \end{bmatrix}$$
$$\tilde{B_{ij}} = \begin{bmatrix} B_j & B_{dj} \\ B_i - L_iD_j + L_iD_i & -L_iD_{dj} \end{bmatrix} \Delta\tilde{B_i} = \begin{bmatrix} \Delta B_j & \Delta B_{dj} \\ -L_i\Delta D_j & -L_i\Delta D_{dj} \end{bmatrix}$$
$$\tilde{C_{ij}} = \begin{bmatrix} H_iC_j & -H_iC_i \end{bmatrix} \Delta\tilde{D_{ij}} = \begin{bmatrix} H_i\Delta D_j & H_i\Delta D_{dj} \end{bmatrix}$$
$$\tilde{D_{ij}} = \begin{bmatrix} H_iD_j - H_iD_i & H_iD_{dj} \end{bmatrix}, \Delta\tilde{C_{ij}} = \begin{bmatrix} H_i\Delta C_j & 0 \end{bmatrix}$$

$$\breve{B}_{fij} = \begin{bmatrix} B_{fj} \\ 0 \end{bmatrix}, \breve{D}_{fij} = \begin{bmatrix} H_iD_{fj} \end{bmatrix}$$

Problem 3. *Given the switched system (3.68) subject to actuator and sensor faults, under the effect of disturbances, noise, and norm bounded uncertainty, design a fault detection filter (FDF) to detect and isolate the faults, such that, the system for average dwell time (ADT) under asynchronous switching is exponentially stable and ensuring* H_∞ *performance,* $\| G_{r\omega} \|_\infty < \gamma_i, \| G_{r\omega} \|_\infty < \gamma_{ij}$ *and* H_- *performance,* $\| G_{r\omega} \|_- > \beta_i$

3.4.2 Solution to the H_-/H_∞ Problem

Main Results

Theorem 3. *Suppose the residual generator (3.70) and (3.71) satisfy the assumptions A1-A4, while the model uncertainty is structured of norm bounded form with* $\Delta^T(t)\Delta(t) \leq 1$ *, then for a given scalar* $\alpha_i \geq 0, \rho_i \geq 0, \mu_1 \geq 1, \mu_2 \geq 1, \beta_i \geq 1,$ *and for any switching signal with ADT,* $\tau_\alpha > \tau_\alpha{}^* = \frac{ln(\mu_1\mu_2)}{\zeta^*}, 0 < \zeta^* < \alpha,$

$$\int_0^\infty (r^T)(r)dt < \gamma_i^2 \int_0^\infty (\omega^T)(\omega)dt \tag{3.72}$$

and

$$\int_0^\infty (r^T)(r)dt < \gamma_{ij}^2 \int_0^\infty (\omega^T)(\omega)dt \tag{3.73}$$

if, there exist symmetric positive definite matrix $P_i > 0, P_{ij} > 0$, for $i \neq j; i, j \in N$, such that the following set of LMIs has a solution;

$$\begin{bmatrix} P_{11ij} & P_{12ij} \\ * & P_{22ij} \end{bmatrix} > 0 \quad , \quad \begin{bmatrix} P_{11i} & P_{12i} \\ * & P_{22i} \end{bmatrix} > 0 \tag{3.74}$$

$$\begin{bmatrix} P_{11j} & P_{12j} \\ * & P_{22j} \end{bmatrix} \leq \mu_1 \begin{bmatrix} P_{11ij} & P_{12ij} \\ * & P_{22ij} \end{bmatrix} \tag{3.75}$$

$$\begin{bmatrix} P_{11ij} & P_{12ij} \\ * & P_{22ij} \end{bmatrix} \leq \mu_2 \begin{bmatrix} P_{11i} & P_{12i} \\ * & P_{22i} \end{bmatrix} \tag{3.76}$$

$$\begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} & \Psi_{14} & {C_i}^T {D_{fi}^{\dagger}}^T {\eta_i}^T + {C_i}^T {D_{fi}^{\perp}}^T {S_i}^T {\eta_i}^T & \Psi_{16} \\ * & \Psi_{22} & \Psi_{23} & \Psi_{24} & -{C_i}^T {D_{fi}^{\dagger}}^T {\eta_i}^T - {C_i}^T {D_{fi}^{\perp}}^T {S_i}^T {\eta_i}^T & \Psi_{26} \\ * & * & \Psi_{33} & \epsilon_i H_i^T J_i & 0 & 0 \\ * & * & * & \Psi_{44} & {D_{di}}^T {D_{fi}^{\dagger}}^T {\eta_i}^T + D_{di}^T {D_{fi}^{\perp}}^T {S_i}^T {\eta_i}^T & 0 \\ * & * & * & * & -I & \Psi_{56} \\ * & * & * & * & * & -\epsilon_i I \end{bmatrix} < 0 \tag{3.77}$$

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & {C_j}^T {D_{fi}^{\dagger}}^T {\eta_i}^T + {C_j}^T {D_{fi}^{\perp}}^T {S_i}^T {\eta_i}^T & \Omega_{16} \\ * & \Omega_{22} & \Omega_{23} & \Omega_{24} & -{C_i}^T {D_{fi}^{\dagger}}^T {\eta_i}^T - {C_i}^T {D_{fi}^{\perp}}^T {S_i}^T {\eta_i}^T & \Omega_{26} \\ * & * & \Omega_{33} & \epsilon_j H_j^T J_j & \Omega_{35} & 0 \\ * & * & * & \Omega_{44} & {D_{dj}}^T {D_{fi}}^{-T} {\eta_i}^T & 0 \\ * & * & * & * & -I & \Omega_{56} \\ * & * & * & * & * & -\epsilon_j I \end{bmatrix} < 0 \tag{3.78}$$

where,

$$\Psi_{11} = A_i^T P_{11i} + C_i^T {D_{fi}^{\dagger}}^T {B_{fi}}^T {P_{12i}}^T - C_i^T {D_{fi}^{\perp}}^T R_i^T P_{12i}^T + P_{11i}A_i + P_{12i}B_{fi}{D_{fi}}^{\dagger}C_i$$
$$- P_{12i}R_iD_{fi}^{\perp}C_i + \epsilon_i G_i^T G_i + \alpha_i P_{11i}$$
$$\Psi_{12} = A_i^T P_{12i} + C_i^T {D_{fi}^{\dagger}}^T {B_{fi}}^T {P_{22i}}^T - C_i^T {D_{fi}^{\perp}}^T R_i^T P_{22i}^T + P_{12i}A_i - P_{12i}B_{fi}{D_{fi}}^{\dagger}C_i$$
$$+ P_{12i}R_iD_{fi}^{\perp}C_i + \alpha_i P_{12i}$$
$$\Psi_{13} = P_{11i}B_i + P_{12i}B_i + \epsilon_i G_i^T H_i$$
$$\Psi_{14} = P_{11i}B_{di} + P_{12i}B_{fi}{D_{fi}}^{\dagger}D_{di} - P_{12i}R_i{D_{fi}}^{\perp}D_{di} + \epsilon_i G_i^T J_i$$
$$\Psi_{16} = P_{11i}E_i + P_{12i}B_{fi}{D_{fi}}^{\dagger}F_i - P_{12i}R_iD_{fi}^{\perp}F_i$$
$$\Psi_{22} = A_i^T P_{22i} - C_i^T {D_{fi}^{\dagger}}^T {B_{fi}}^T {P_{22i}}^T + C_i^T {D_{fi}^{\perp}}^T R_i^T P_{22i}^T + P_{22i}A_i$$
$$- P_{22i}B_{fi}{D_{fi}}^{\dagger}C_i + +P_{12i}R_iD_{fi}^{\perp}C_i + \alpha_i P_{22i}$$
$$\Psi_{23} = {P_{12i}}^T B_i + P_{22i}B_i$$
$$\Psi_{14} = P_{12i}^T B_{di} + P_{22i}B_{fi}{D_{fi}}^{\dagger}D_{di} - P_{22i}R_i{D_{fi}}^{\perp}D_{di}$$
$$\Psi_{26} = P_{12i}E_i + P_{22i}B_{fi}{D_{fi}}^{\dagger}F_i - P_{22i}R_iD_{fi}^{\perp}F_i$$
$$\Psi_{33} = \epsilon_i H_i^T H_i - \gamma_i^2, \Psi_{44} = \epsilon_i J_i^T J_i - \gamma_i^2$$
$$\Psi_{56} = -\eta_i D_{fi}^{\dagger}F_i - \eta_i S_i D_{fi}^{\perp}F_i$$
$$\Omega_{11} = A_j^T P_{11ij} + C_j^T {D_{fi}^{\dagger}}^T {B_{fi}}^T {P_{12ij}}^T - C_j^T {D_{fi}^{\perp}}^T R_i^T P_{12ij}^T + P_{11ij}A_j$$
$$+ P_{12ij}B_{fi}{D_{fi}}^{\dagger}C_j - P_{12ij}R_iD_{fi}^{\perp}C_j + \epsilon_j G_j^T G_j + \rho_i P_{11ij}$$
$$\Omega_{12} = A_j^T P_{12ij} + C_j^T {D_{fi}^{\dagger}}^T {B_{fi}}^T {P_{22ij}}^T - C_j^T {D_{fi}^{\perp}}^T R_i^T P_{22ij}^T + P_{12ij}A_i - P_{12ij}B_{fi}{D_{fi}}^{\dagger}C_i$$
$$+ P_{12ij}R_iD_{fi}^{\perp}C_i + \rho_i P_{12ij}$$
$$\Omega_{13} = P_{11ij}B_j + P_{12ij}B_i + P_{12ij}B_{fi}{D_{fi}}^{\dagger}D_j - P_{12ij}R_i{D_{fi}}^{\perp}D_j - P_{12ij}B_{fi}{D_{fi}}^{\dagger}D_i$$
$$+ P_{12ij}R_i{D_{fi}}^{\perp}D_i + \epsilon_j G_j^T H_j$$

$$\Omega_{14} = P_{11ij}B_{dj} + P_{12ij}B_{fi}{D_{fi}}^{\dagger}D_{dj} - P_{12ij}R_iD_{fi}{}^{\perp}D_{dj} + \epsilon_j G_j^T J_j$$

$$\Omega_{16} = P_{11ij}E_j + P_{12ij}B_{fi}{D_{fi}}^{\dagger}F_j - P_{12ij}R_iD_{fi}^{\perp}F_j$$

$$\Omega_{22} = A_i^T P_{22ij} - C_i^T {D_{fi}}^{\dagger^T} {B_{fi}}^T {P_{22ij}}^T + C_i^T {D_{fi}^{\perp}}^T R_i^T P_{22ij}^T + P_{22ij}A_i$$

$$- P_{22ij}B_{fi}{D_{fi}}^{\dagger}C_i + +P_{12ij}R_iD_{fi}^{\perp}C_i - \rho_i P_{22ij}$$

$$\Omega_{23} = P_{12ij}B_j + P_{22ij}B_i + P_{22ij}B_{fi}{D_{fi}}^{\dagger}D_j - P_{22ij}R_i{D_{fi}}^{\perp}D_j - P_{22ij}B_{fi}{D_{fi}}^{\dagger}D_i$$

$$+ P_{22ij}R_i{D_{fi}}^{\perp}D_i$$

$$\Omega_{24} = {P_{12ij}}^T B_{dj} + P_{22ij}B_{fi}D_{fi}^{-1}D_{dj}$$

$$\Omega_{26} = P_{12ij}E_j + P_{22ij}B_{fi}{D_{fi}}^{\dagger}F_j - P_{22ij}R_iD_{fi}^{\perp}F_j$$

$$\Omega_{33} = \epsilon_j H_j^T H_j - \gamma_{ij}^2$$

$$\Omega_{35} = D_j^T {D_{fi}}^{\dagger^T} {\eta_i}^T + D_j^T {D_{fi}^{\perp}}^T {S_i}^T {\eta_i}^T - D_i^T {D_{fi}}^{\dagger^T} {\eta_i}^T - D_i^T {D_{fi}^{\perp}}^T {S_i}^T {\eta_i}^T$$

$$\Omega_{44} = \epsilon J_j^T J_j - {\gamma_{ij}}^2 I,$$

$$\Omega_{45} = D_{dj}^T {D_{fi}}^{\dagger^T} {\eta_i}^T + D_{dj}^T {D_{fi}^{\perp}}^T {S_i}^T {\eta_i}^T$$

$$\Omega_{56} = -\eta_i D_{fi}^{-1}F_j - \eta_i S_i D_{fi}^{\perp}F_j$$

Moreover, parameters of detection filter are derived as

$$L_i = -B_{fi}{D_{fi}}^{\dagger} + R_i{D_{fi}}^{\perp}, H_i = \beta_i({D_{fi}}^{\dagger} + S_i{D_{fi}}^{\perp})$$

where $R_i \in R^{n\times(m-q)}$ *and* $S_i \in R^{q\times(m-q)}$*, are additional variables that are introduced to provide more degree of freedom for* L_i *and* H_i*.*

Proof. We require that $\|F_iG_{fi}\|_- \geq \beta_i$ and also we know that

$$F_i(s)G_{fi}(s) = (A_i + L_iC_i, B_{fi} + L_iD_{fi}, H_iC_i, H_iD_{fi}) \in \Re H_\infty^{q\times m}$$

To achieve the above mentioned objective, we should have $F_iG_{fi} = \eta_i I$, so that $\|F_iG_{fi}\|_- \geq \eta_i \; \forall i \in \{1, 2, ...N\}$ This can be achieved easily by setting

$$B_{fi} + L_iD_{fi} = 0 \text{ and } H_iD_{fi} = \eta_i I_q$$

From these two equations, we can find $L_i = -B_{fi}{D_{fi}}^{\dagger} + R_i{D_{fi}}^{\perp}$, and$H_i = \beta_i({D_{fi}}^{\dagger} + S_i{D_{fi}}^{\perp})$ In addition, by aforementioned setting of terms isolation framework is achieved [5] Now, for the desired L_i and H_i the only remaining part of the problem is to find out H_∞ norm of $F_iG_{di}, \forall i \in \{1, 2, ...N\}$. To this end, we use Lemma 1 under asynchronous paradigm during matched and unmatched period. LMIs (3.74)- (3.76) are necessary conditions for stability of switched systems and are derived in [93] with details. Next we derive the results of (3.77) and (3.78).

Stability Analysis during Matched Period:

Using Lemma 3 (Appendix), here, $u(t) = \omega(t)$ and $y(t) = r(t)$

$$V_i(\tilde{x}(t)) \leq \mu V_j(\tilde{x}(t)) \tag{3.79}$$

$$\dot{V}_i(\tilde{x}(t)) \leq -\alpha V_i(\tilde{x}(t)) - r^T r(t) + \gamma_i^2 \omega(t)^T \omega(t) \tag{3.80}$$

Considering the following Lyapunov function, during this time

$$V_i(\tilde{x}(t)) = \tilde{x}^T(t) P_i \tilde{x}(t) \tag{3.81}$$

Differentiating (3.81)

$$\dot{V}_i(\tilde{x}(t)) = \dot{\tilde{x}}^T(t) P_i \tilde{x}(t) + \tilde{x}^T(t) P_i \dot{\tilde{x}}(t) \tag{3.82}$$

after substituting $r(t)$ from (3.70) and (3.81), (3.82), in (3.80), and also

$$\breve{A} = (\tilde{A}_i + \Delta\tilde{A}_i), \breve{B} = (\tilde{B}_i + \Delta\tilde{B}_i)$$
$$\breve{C} = (\tilde{C}_i + \Delta\tilde{C}_i), \breve{D} = (\tilde{D}_i + \Delta\tilde{D}_i)$$

following inequality is obtained

$$\begin{aligned} &\dot{\tilde{x}}^T(t) P_i \tilde{x}(t) + \tilde{x}^T(t) P_i \dot{\tilde{x}}(t) + \left[\breve{C}_i \tilde{x}(t) + \breve{D}_i \omega(t)\right]^T \left[\breve{C}_i \tilde{x}(t) + \breve{D}_i \omega(t)\right] \\ &\leq -\alpha \tilde{x}^T(t) P_i \tilde{x}(t) + \gamma_i^2 \omega(t)^T \omega(t) \end{aligned} \tag{3.83}$$

Further, after substituting the expression for $\dot{\tilde{x}}(t)$ from (3.70) in (3.83),

$$\begin{aligned} &\left[\breve{A}_i \tilde{x}(t) + \breve{B}_i \omega(t)\right]^T P_i \tilde{x}(t) + \tilde{x}^T(t) P_i \left[\breve{A}_i \breve{x}(t) + \breve{B}_i \omega(t)\right] \\ &+ (\tilde{x}^T(t) \breve{C}_i^T + \omega^T(t) \breve{D}_i^T)(\breve{C}_i \tilde{x}(t) + \breve{D}_i \omega(t)) \\ &\leq -\alpha \tilde{x}^T(t) P_i \tilde{x}(t) + \gamma_i^2 \omega(t)^T \omega(t)) \end{aligned} \tag{3.84}$$

(3.84) can be written easily in following form

$$\begin{aligned}&\left[\tilde{x}^T(t)\breve{A}_i^T P_i + \omega^T(t)\breve{B}_i^T P_i\right]\tilde{x}(t) + \tilde{x}^T(t)P_i\breve{A}_i\tilde{x}(t) + \tilde{x}^T(t)P_i\breve{B}_i\omega(t) + \tilde{x}^T(t)\breve{C}_i^T\breve{C}_i\tilde{x}(t)\\&+ \tilde{x}^T(t)\breve{C}_i^T\breve{D}_i\omega(t) + \omega^T(t)\breve{D}_i^T\breve{C}_i\tilde{x}(t) + \omega^T(t)\breve{D}_i^T\breve{D}_i\omega(t)) + \alpha\tilde{x}^T(t)P_i\tilde{x}(t)\\&- {\gamma_i}^2\omega^T(t)\omega(t) \leq 0\end{aligned} \tag{3.85}$$

Further, the above inequality can be written as

$$\begin{bmatrix}\tilde{x}^T(t) & \omega^T(t)\end{bmatrix}\mathbf{M}\begin{bmatrix}\tilde{x}(t)\\ \omega(t)\end{bmatrix} \leq 0 \tag{3.86}$$

Where,

$$\mathbf{M} = \begin{bmatrix}\breve{A}_i{}^T P_i + P_i\breve{A}_i + \breve{C}_i{}^T\breve{C}_i + \alpha P_i & P_i\breve{B}_i + \breve{C}_i{}^T\breve{D}_i\\ * & \breve{D}_i{}^T\breve{D}_i - \gamma_i^2 I\end{bmatrix}$$

for (3.86) to hold, it is required that

$$\mathbf{M} < 0 \tag{3.87}$$

After Schur's compliment ([102]) is applied to (3.87), we get

$$\begin{bmatrix}\breve{A}_i{}^T P_i + P_i\breve{A}_i + \alpha P_i & P_i\breve{B}_i & \breve{C}_i{}^T\\ * & -\gamma_i^2 I & \breve{D}_i{}^T\\ * & * & -I\end{bmatrix} < 0 \tag{3.88}$$

To separate the uncertainties terms, splitting the LMI (3.88)

$$\begin{bmatrix}\tilde{A}_i{}^T P_i + P_i\tilde{A}_i + \alpha P_i & P_i\tilde{B}_i & \tilde{C}_i{}^T\\ * & -\gamma_i^2 I & \tilde{D}_i{}^T\\ * & * & -I\end{bmatrix} + \begin{bmatrix}\Delta\tilde{A}_i{}^T P_i + P_i\Delta\tilde{A}_i & P_i\Delta\tilde{B}_i & \Delta\tilde{C}_i{}^T\\ * & 0 & \Delta\tilde{D}_i{}^T\\ * & * & 0\end{bmatrix} < 0 \tag{3.89}$$

second matrix in above inequality can be written into following following form

$$\begin{bmatrix} \Delta\tilde{A_i}^T P_i + P_i\Delta\tilde{A}_i & P_i\Delta\tilde{B}_i & \Delta\tilde{C_i}^T \\ * & 0 & \Delta\tilde{D_i}^T \\ * & * & 0 \end{bmatrix} = \begin{bmatrix} \bar{E}1 \\ \bar{E}2 \\ \bar{E}3 \\ 0 \\ 0 \\ 0 \\ -H_iF \end{bmatrix} \Delta(t) \begin{bmatrix} 0 & G & 0 & H & J & 0 & 0 \end{bmatrix}$$

$$+ [\begin{bmatrix} \bar{E}1 \\ \bar{E}2 \\ \bar{E}3 \\ 0 \\ 0 \\ 0 \\ -H_iF \end{bmatrix} \Delta(t) \begin{bmatrix} 0 & G & 0 & H & J & 0 & 0 \end{bmatrix}]^T \tag{3.90}$$

According to Lemma 2, we know that (3.89) holds if there exists $\epsilon > 0$ so that

$$\begin{bmatrix} \tilde{A_i}^T P_i + P_i\tilde{A}_i + \alpha P_i & P_i\tilde{B}_i & \tilde{C_i}^T \\ * & -\gamma_i^2 I & \tilde{D_i}^T \\ * & * & -I \end{bmatrix} + \frac{1}{\epsilon} \begin{bmatrix} \bar{E}1 \\ \bar{E}2 \\ \bar{E}3 \\ 0 \\ 0 \\ 0 \\ -H_iF \end{bmatrix} \begin{bmatrix} \bar{E}1 \\ \bar{E}2 \\ \bar{E}3 \\ 0 \\ 0 \\ 0 \\ -H_iF \end{bmatrix}^T \tag{3.91}$$

$$+ \epsilon \begin{bmatrix} 0 & G & 0 & H & J & 0 & 0 \end{bmatrix}^T \begin{bmatrix} 0 & G & 0 & H & J & 0 & 0 \end{bmatrix} < 0$$

Finally, by applying Schur's complement again, following LMI is obtained

$$\begin{bmatrix} \psi & P_i\tilde{B} + \epsilon\bar{G}^T\bar{H} & \tilde{C}_i^T & P_i\bar{E} \\ * & -\gamma_i^2 I + \epsilon\bar{H}^T\bar{H} & \tilde{D}_i^T & 0 \\ * & * & -I & -H_iF \\ * & * & * & -\epsilon I \end{bmatrix} < 0 \tag{3.92}$$

where ,$\psi = \tilde{A_i}^T P_i + P_i\tilde{A}_i + \alpha P_i + \epsilon\bar{G}^T\bar{G}, \bar{G} = [G0], \bar{H} = [HJ], \bar{E}^T = [E^T(-L_iF)^T]$

Then, substituting, L_i and H_i in (3.92), LMI (3.77) of the Theorem 1 is obtained.

Stability Analysis during Unmatched Period:

During unmatched period,

$$V_{ij}(\tilde{x}(t)) \leq \mu v_j(\tilde{x}(t)) \tag{3.93}$$

and

$$\dot{V}_{ij}(\tilde{x}(t)) \leq \rho V_{ij}(\tilde{x}(t)) - r^T r(t) + \gamma_i^2\omega(t)^T\omega(t) \tag{3.94}$$

where $i \neq j$ and $i, j \in N$

$$\dot{V}_{ij}(\tilde{x}(t)) + r^T r(t) \leq \rho v_{ij}(\tilde{x}(t)) + \gamma_i^2\omega(t)^T\omega(t) \tag{3.95}$$

During unmatched period, the following Lyapunov function is used

$$V_{ij}(\tilde{x}(t)) = \tilde{x}^T(t)P_{ij}\tilde{x}(t) \tag{3.96}$$

To derive further the results of Theorem, we apply the same procedure as it is done earlier for matched period. To this end, we skip the next steps which result in inequality (3.78) of theorem.

Now, the proof is completed which ensures the sensitivity level β_i , disturbance attenuation level γ_i $\forall i \in \{1, 2, ...N\}$ and isolation capability of the derived filter. □

3.4.3 Adaptive Threshold Computation and Residual Evaluation

After successful residual generation, the next step is to evaluate further the residual signal. The importance of this step is due to the fact that residual may be nonzero even if there is no fault in the system. In literature, there exist many types of residual evaluation function. In this work following residual evaluation function is used which is based on RMS energy of the residual signal, that is given as

$$J_{RMS} = \parallel r(t) \parallel_{RMS} = (\frac{1}{T}\int_{t}^{t+T} \parallel r(\tau) \parallel^2 d\tau)^{\frac{1}{2}} \tag{3.97}$$

where, T is the evaluation window.

Along with residual evaluation function, the threshold computation is also required for efficient detection of faults. Threshold value is the maximum influence of unknown inputs (disturbances, noises) and model uncertainties on the residual signal in the absence of faults. Threshold can also be of different types like fixed, adaptive, or dynamic [103, 90, 95] depending on application under consideration. In this research the following adaptive threshold is employed,

$$J_{adap.th,\sigma(t)RMS,2} = \frac{\gamma_\sigma(t)}{\sqrt{T}}(\delta_{d,2,\sigma(t)} + \|u\|_{2,\sigma(t)}) \tag{3.98}$$

where, $\gamma_\sigma(t)$ is mode dependent robustness factor, $\delta_{d,2,\sigma(t)}$ is norm bounded disturbance acting on the corresponding mode which can be found set off-line and $\|u\|_{2,\sigma(t)}$ is input which is time dependent parameter, computed on-line. In this way to improve fault detection rate and reduce false alarm rate, individual mode based robustness factors are considered, instead of global worst-case parameters and effect of input on residual is also taken into account at residual evaluation stage.

3.4.4 Application to the Case Study: Buck-boost Converter

In this section simulation results for the case study of Buck-boost converter discussed in Section 3.3. The Simulation time is setup for 30s, such that, subsystem 1 is activated when $\sigma(t) = 1$ and subsystem 2 when $\sigma(t) = 0$. In this work, based on the proposed FDI approach we are able to detect and isolate two faults occurring at a time. Details of temporal switching behaviour of subsystems, filters and faults can be seen in Fig. 3.19.

Results and Discussion

Next, we discuss the results, based on the dynamics of Buck-boost converter, presented in Section 3.3. Uncertainties matrices are given below

$$E_1 = \begin{bmatrix} 0.2 \\ 0.35 \\ 0.31 \end{bmatrix}, F_1 = \begin{bmatrix} 0.1 \\ 0.25 \\ 0.19 \end{bmatrix}, G_1 = \begin{bmatrix} 0.2 & 0.3 & 0.22 \end{bmatrix} H_1 = \begin{bmatrix} 0.2 \end{bmatrix}, J_1 = \begin{bmatrix} 0.3 & 0.2 \end{bmatrix},$$

$$E_2 = \begin{bmatrix} 0.3 \\ 0.25 \\ 0.16 \end{bmatrix}, F_2 = \begin{bmatrix} 0.2 \\ 0.25 \\ 0.18 \end{bmatrix}, G_2 = \begin{bmatrix} 0.3 & 0.2 & 0.18 \end{bmatrix} H_2 = \begin{bmatrix} 0.3 \end{bmatrix}, J_2 = \begin{bmatrix} 0.2 & 0.35 \end{bmatrix}$$

By solving LMIs (3.74)-(3.78) of Theorem 3, the system is simulated as in Fig. 3.19. The residual signals for the system in absence of faults are depicted in Fig. 3.20. It is clear that residual is non-zero in the absence of faults, which may generate false alarm. To this end, residual signal is evaluated and adaptive threshold is used in Fig. 3.21.

The residual signals for the system in the presence of faults are depicted in Fig. 3.22. These results are also evaluated and compared with threshold in Fig. 3.23 In Fig. 3.22 and Fig. 3.23 , it is easy to see that $f_1(t)$ affects only the $r_1(t)$ whereas $f_2(t)$ has influence exclusively to $r_2(t)$. In this way, not only both faults are successfully detected but also isolated (located) for uncertain switched system.

Finally, to avoid false alarms, residuals are evaluated by RMS and adaptive thresholds are designed according to 3.97 and 3.98, which are depicted in Fig.3.21 and Fig.3.23 . For residual evaluation window size T is set to 10. Fault is detected properly when evaluated residuals cross their respective adaptive thresholds values just after the occurrence of faults at time 3s and 8s.

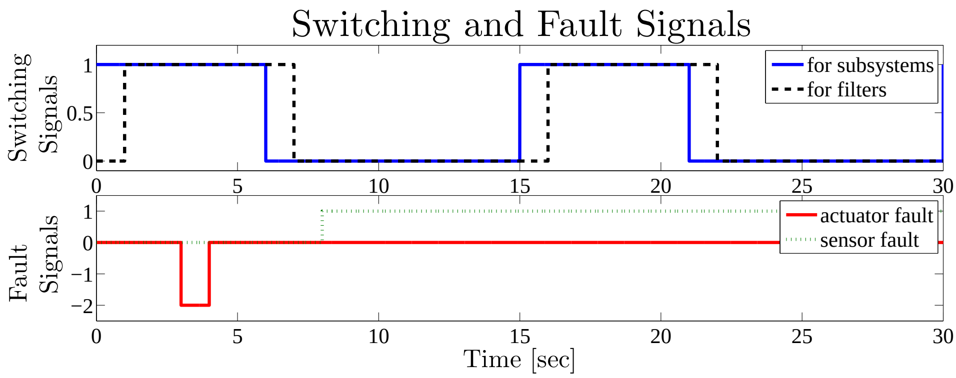

Figure 3.19: Switching signals $\sigma(t)$ for subsystems and Switching signal $\sigma'(t)$ for filters: Actuator and sensor faults

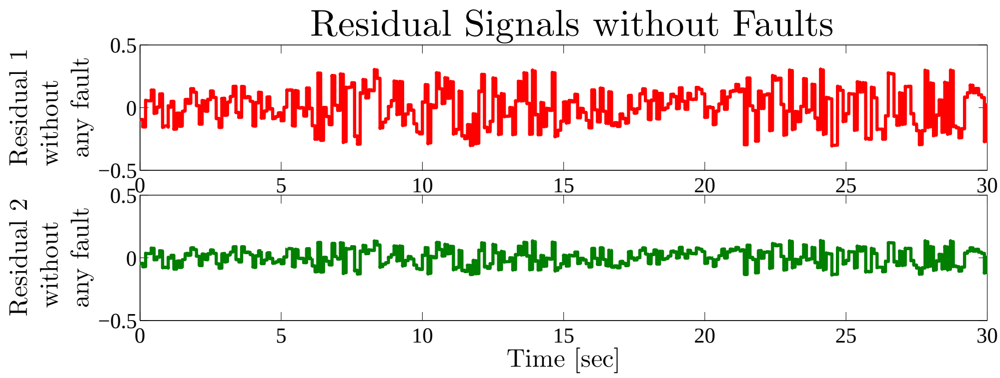

Figure 3.20: Residual signal 1, $r_1(t)$ without any fault : Residual signal 2, $r_2(t)$ without any fault

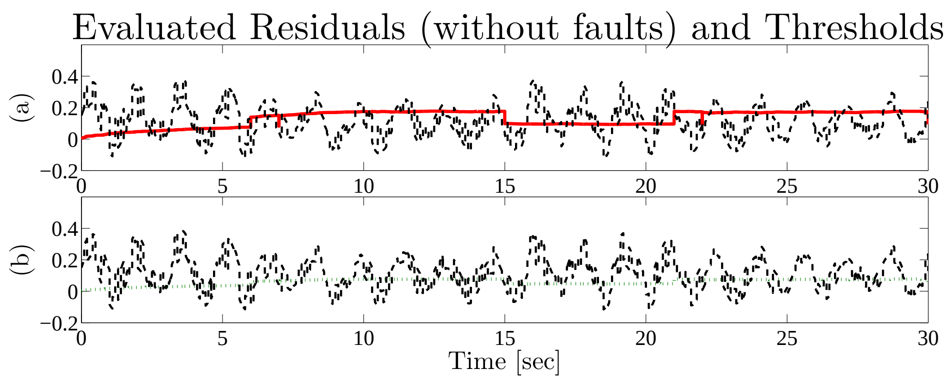

Figure 3.21: (a): Evaluated residual signal 1 and thresholds (b): Evaluated residual signal 2 and thresholds

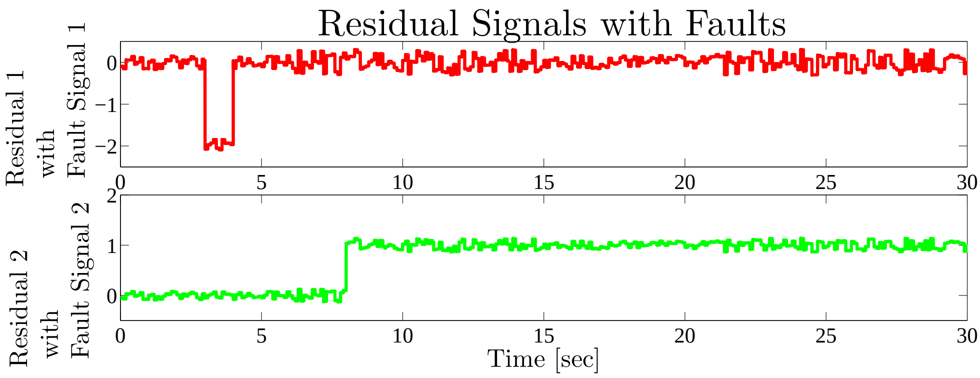

Figure 3.22: Residual signal 1 with fault : Residual signal 2 with fault

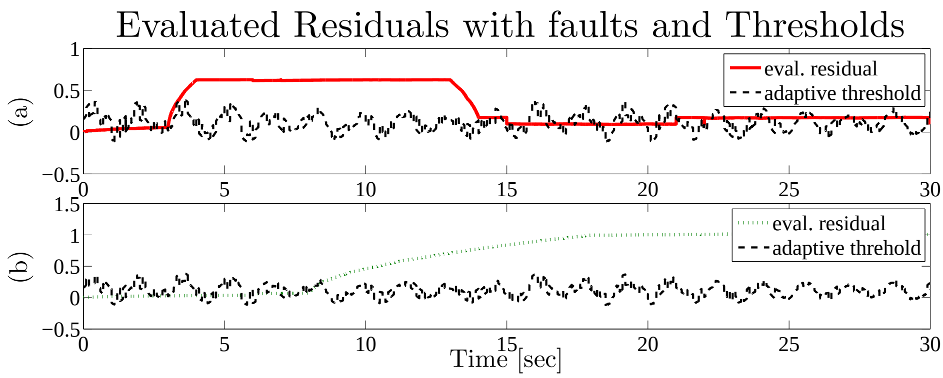

Figure 3.23: (a): Evaluated residual signal 1 and thresholds (b): Evaluated residual signal 2 and thresholds

3.5 Summary

In the first part of the chapter, the problem of fault detection, in the presence of disturbances and noise, for continuous-time linear switched control systems is addressed. The residual generator is proposed which is based on asynchronously switching filters. To address the issue, fault detection filter problem is formulated as H_∞ filtering problem. In proposed H_∞ technique, residual is generated such that it is robust against process disturbances and measurement noise. To deal with the major issue of asynchronous switching, during matched and unmatched time of switched systems, a piecewise Lyapunov function along with average dwell time scheme is employed, and sufficient conditions are derived in terms of linear matrix inequalities. On the basis of proposed results, fault detection scheme is designed for battery converter unit of hybrid electric vehicle.

The obtained results are encouraging to apply further the technique to whole drive train of HEVs to improve the performance, reliability, and safety of the vehicles.

In the second part of the chapter, the problem of fault detection and isolation (FDI) for continuous-time switched system has been addressed. To address the issue, fault detection and isolation problem is formulated as $H_- \ H_\infty$ filtering problem. In proposed $H_- \ H_\infty$ technique, residual is generated such that it is robust against process disturbances and measurement noise. On the basis of proposed results, fault detection and isolation scheme is designed for Buck-boost converter. The obtained results are encouraging to apply further the technique to whole drive train of HEVs to improve the performance, reliability, and safety of the vehicles.

The last part of the chapter deals with, FDI problem for uncertain switched systems. For FDI, model based technique has been proposed for the available state space model with the assumption of norm-bounded uncertainties. Solution is provided in the form of a mixed H_-/H_∞ fault detection filter. ADT constraint has been considered while employing piecewise Lyapunov function and the results are derived in the form of LMIs. To improve the fault detection adaptive threshold is employed. Application of results for Buck-boost converter shows the effectiveness of proposed strategy.

Chapter 4

Fault Estimation and Tolerance in Switched Systems

In this chapter, problem of fault estimation and fault tolerance for switched systems under asynchronous switching is discussed. Unknown input observer is designed to estimate actuator and sensor faults while reconfiguration strategy is employed to compensate the faults.

4.1 Introduction

This chapter presents an active fault tolerance approach to compensate actuator and sensor faults in switched systems under the effect of disturbances. Further, to deal with practical case of event-based switching, observers are assumed to be switching asynchronously with corresponding modes of the system. To tolerate faults, firstly robust model-based (software redundant) switching unknown input observer are designed for each mode of the system for estimation of the faults. To develop the strategy, observer-based problem is solved via H_∞ optimization with the help of linear matrix inequalities (LMI) formulation. By using a piece-wise Lyapunov function, under average dwell-time constraint, sufficient conditions are derived in the form of linear matrix inequalities. At the end, a switched system example is given to illustrate the design procedure and the validity of the proposed integrated design approach.

Basic concepts related to FDD, FTC. and switched systems have been discussed in detail in Chapter 2. Now, we present a survey on fault estimation and fault tolerant control for switched systems. Work in [104] presents the technique for closed loop stability in the faulty situation of non-linear switched system. To ensure stability of non-linear switched system under two types of faults is presented in [105]. Faults during dwell time and faults in switching sequence are considered therein. In [106] two techniques based on data-driven fault diagnosis are presented for Tennessee Eastman (TE) benchmark process. In [107], a technique is presented for fault estimation and tolerance for actuator faults in discrete switched systems. In [108] an adaptive FE algorithm is proposed for estimating constant and time-varying faults in class of switched systems with time-varying delays. On the basis of FE an FTC is developed for closed loop stability after occurrence of fault. Further, in model-based FTC research domain, [43] proposed the sliding mode observer-based FTC based originally on FDI. A motivating contribution, [46] presents an integrated approach for FE and active FTC in time-varying descriptor systems. The proposed method can simultaneously estimate states, actuator and sensor faults firstly and then an FTC is proposed.

The rest of the chapter is organized as follows: Next, research contribution of this chapter is presented. In Section 4.2, problem is formulated for fault estimation and fault tolerance of switched systems. In Section 4.3, solution is proposed to estimate the actuator and sensor faults in asynchronous switching scenario. In Section 4.4, a scheme is developed for fault tolerance. In Section 4.5, application results are demonstrated to show the effectiveness of the approach, followed by chapter summary in Section 4.6.

Research Contribution

This work, is continuity of our results [101], with the overall objective: to explore the complete design chain of fault management system. In [101], a solution for fault detection and isolation was presented. Here the objective is to design FTC on the basis of FE, directly. Event based UIO is designed in asynchronous switching paradigm. The observer estimates the actuator and sensor faults successfully. Based on the fault estimations, closed loop stability is ensured for fault tolerance,in terms of LMIs, using H_∞ filtering. Schemes are designed under the effect of disturbances and noises.

4.2 Problem Formulation: Fault Estimation and Tolerance in Switched Systems

System description

Consider the following class of continuous-time switched systems

$$\begin{aligned} \dot{x}(t) &= A_{\sigma(t)}x\,(t) + B_{\sigma(t)}u\,(t) + B_{d\sigma(t)}d\,(t) + B_{fa\sigma(t)}f_a(t) \\ y(t) &= C_{\sigma(t)}x(t) + D_{fs\sigma(t)}f_s(t) \end{aligned} \tag{4.1}$$

where $x(t) \in R^n$ is a state vector, $u(t) \in R^r$ is a control input vector, $y(t) \in R^m$ is an output vector, $d(t) \in R^p$ is an unknown input (disturbances, noise) vector, $f_a(t) \in R^{q_1}$and $f_s(t) \in R^{q_2}$ are fault vectors, $\sigma(t)$ is a switching signal which is a piecewise constant function $\sigma : [0, \infty) \longrightarrow \rho$. Such a function σ has a finite number of switching times and takes a constant value on every interval between two consecutive switching times. The role of $\sigma(t)$ is to specify, at each instant t, the index $\sigma(t) \in \rho$ of the active subsystem. Also, $A_{\sigma(t)}, B_{\sigma(t)}, C_{\sigma(t)}, B_{d\sigma(t)}, B_{fa\sigma(t)}, D_{fs\sigma(t)}$ are the systems, disturbances and fault coupling matrices with appropriate dimensions. We denote the association of these matrices with particular switching signal instant $\sigma(t) = i$ by $A_{\sigma(t)} = A_i$, where $i = 1, 2, ...N$, number of subsystems involved.

Problem 4. *Given the switched system (1) subject to actuator and sensor sensor faults of constant type, under the effect of disturbances, design an active FTC approach based on FE to ensure that the closed loop is stable with H_∞ performance $\parallel G_{y_{cmp}d} \parallel_\infty < \gamma_\sigma$.*

4.3 Solution to the Fault Estimation Problem: Unknown input observer

In order to estimate the actuator and sensor faults, the fault vectors $f_a(t)$ and $f_s(t)$ are augmented with state of the 4.1 as follows

$$\begin{aligned} \dot{\bar{x}}(t) &= A_{\sigma(t)}\bar{x}(t) + \bar{B}_{\sigma'(t)}u(t) + \bar{D}_{\sigma}d(t) \\ \hat{y}(t) &= \bar{C}_{\sigma(t)}\bar{x}(t) + \bar{D}_{\sigma(t)}d(t) \end{aligned} \tag{4.2}$$

where,

$$\bar{x}(t) = \begin{bmatrix} x(t) \\ f_a(t) \\ f_s(t) \end{bmatrix}, \bar{A}_\sigma = \begin{bmatrix} A_\sigma & B_{fa\sigma} 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\bar{B}_\sigma = \begin{bmatrix} B_\sigma \\ 0 \\ 0 \end{bmatrix}, \bar{D}_\sigma = \begin{bmatrix} D_{d\sigma} \\ 0 \\ 0 \end{bmatrix}, \bar{C}_\sigma = \begin{bmatrix} C_\sigma & 0 & D_{fs\sigma} \end{bmatrix}$$

To estimate the fault vectors, following UIO is designed,

$$\dot{z_\sigma} = M_\sigma z + G_\sigma \bar{B}_\sigma u + L_\sigma y$$
$$\hat{x}_\sigma = z + H_\sigma y \tag{4.3}$$

To estimate and tolerate faults in switched systems, observers are designed corresponding to each mode of operation. In switching of observers to their corresponding modes practically, there exists the phenomenon of asynchronous switching between them. The phenomenon has been studied in detail in Chapter 2.

Fault Estimation during Matched Period

Let, the estimation error, $e_\sigma = \bar{x}_\sigma - \hat{x}_\sigma$ then, during matched period

$$\begin{aligned} \dot{e}_i &= \dot{\bar{x}}_i - \dot{\hat{x}}_i \\ &= (\Xi_i \bar{A}_i - L_{1i} \bar{C}_i) e_i + (\Xi_i \bar{A}_i - L_{1i} \bar{C}_i - M_i) z \\ &+ (\Xi_i - G_i) \bar{B}_i u + [(\Xi_i \bar{A}_i - L_{1i} \bar{C}_i) H_i - L_{2i}] y + (\Xi_i \bar{D}_i) d \end{aligned} \tag{4.4}$$

where, $\Xi_i = I_{n+q1+q2} - H_i \bar{C}$ and $L_i = L_{1i} + L_{2i}$
Necessary conditions for the stability and unbiasedness of the error dynamics 4.4 are

$$M_i \ is \ Hurwitz \tag{4.5}$$
$$(\Xi_i \bar{A}_i - L_{1i} \bar{C}_i - M_i) = 0 \tag{4.6}$$
$$\Xi_i - G_i = 0 \tag{4.7}$$
$$(\Xi_i \bar{A}_i - L_{1i} \bar{C}_i) H_i - L_{2i} = 0 \tag{4.8}$$

By satisfying, the conditions (4.5)-(4.8), the error dynamics 4.4 takes the form

$$\dot{e}_i = (\Xi_i \bar{A}_i - L_{1i}\bar{C}_i)e_i + (\Xi_i \bar{D}_i)d \tag{4.9}$$

Fault Estimation during Unmatched Period

As per already defined, estimation error, $e_\sigma = \bar{x}_\sigma - \hat{x}_\sigma$, during unmatched period

$$\begin{aligned}\dot{e}_{ij} &= \dot{\bar{x}}_j - \dot{\hat{x}}_i \\ &= (\Xi_j \bar{A}_j - L_{1i}\bar{C}_j)e_{ij} + (\Xi_j \bar{A}_j - L_{1i}\bar{C}_j - M_i)z \\ &+ (\Xi_j - G_i)\bar{B}_j u + [(\Xi_j \bar{A}_j - L_{1i}\bar{C}_j)H_i - L_{2i}]y + \\ &(\Xi_j \bar{D}_j)d\end{aligned} \tag{4.10}$$

where, $\Xi_j = I_{n+q1+q2} - H_i\bar{C}_j$ and $L_i = L_{1i} + L_{2i}$

Necessary conditions for the stability and unbiasedness of the error dynamics 4.10 are

$$M_i \ is \ Hurwitz \tag{4.11}$$

$$(\Xi_j \bar{A}_j - L_{1i}\bar{C}_j - M_i) = 0 \tag{4.12}$$

$$\Xi_j - G_i = 0 \tag{4.13}$$

$$(\Xi_j \bar{A}_j - L_{1i}\bar{C}_j)H_i - L_{2i} = 0 \tag{4.14}$$

By satisfying, the conditions (4.11)-(4.14), the error dynamics 4.10 takes the form

$$\dot{e}_{ij} = (\Xi_j \bar{A}_j - L_{1i}\bar{C}_j)e_{ij} + (\Xi_j \bar{D}_j)d \tag{4.15}$$

4.4 Solution to the FTC Problem: Reconfiguration

After the fault occurs, based on estimation of fault, next step is to reconfigure the control law which may be activated to compensate for the fault. In faulty case, control law is given as

$$u_{\sigma(t)} = u_{nom,\sigma(t)}(t) + u_{add,\sigma(t)} \tag{4.16}$$

where, $u_{nom,\sigma(t)}(t)$ is nominal control law utilized in normal case, let $u_{nom,\sigma(t)}(t) = Kx_\sigma$, whereas $u_{add,\sigma(t)}$ is the additional control law in case of fault occurrence. Now, 4.1 is written in the form

$$\begin{aligned} \dot{x}(t) &= A_{\sigma(t)}x(t) + B_{\sigma(t)}Kx_{\sigma} + B_{\sigma(t)}u_{add,i}(t) + B_{d\sigma(t)}d(t) + B_{fa\sigma(t)}f_a(t) \\ y(t) &= C_{\sigma(t)}x(t) + D_{fs\sigma(t)}f_s(t) \end{aligned} \tag{4.17}$$

Additional control law is computed online, by using the estimates of faults, such that the response of faulty system is as close as possible to the fault-free system behaviour, that is

$$B_{\sigma(t)}u_{add,\sigma(t)} + B_{fa\sigma(t)}\hat{f}_a(t) = 0.$$

In this way,

$$u_{add,\sigma(t)} = -(B^T_{\sigma(t)}B_{\sigma(t)})^{-1}B^T_{\sigma(t)}B_{fa\sigma(t)}\hat{f}a(t) \tag{4.18}$$

Let, $K_{fa\sigma} = -(B^T_{\sigma(t)}B_{\sigma(t)})^{-1}B^T_{\sigma(t)}B_{fa\sigma(t)}\hat{f}a(t)$ then, $u(t) = K_{\sigma}\hat{x_o}$, where, $K_{\sigma} = [K_{x\sigma}\ K_{fa\sigma}\ 0]$ and $\hat{x_o} = [x\ \hat{f_a}\ \hat{f_s}]^T$ whereas to compensate for sensor fault, following configuration law is used

$$y_{cmp} = y - D_{fs\sigma(t)}\hat{f}_s(t) \tag{4.19}$$

, where, y_{cmp} is sensor fault compensated output. Now system 4.1 can be written as

$$\dot{x}(t) = (A_{\sigma(t)} + B_{\sigma(t)}K_{x\sigma})x - B_{\sigma(t)}K_{x\sigma}e + B_{d\sigma(t)}d$$

$$y_{cmp} = y - D_{fs\sigma(t)}\hat{f}_s(t) \tag{4.20}$$

Fault Tolerance during Matched Period

During the matched period, ith subsystem and ith observer are in operation, see Fig.2.7. To formulate the problem into H_∞ filtering for $y_{cmp}(t)$, during matched period, from 4.9 and 4.20 we augment the switched system and UIO, into the following compact representation

$$\dot{\tilde{x}}(t) = \tilde{A}_i \tilde{x}(t) + \tilde{B}_i d(t)$$
$$y_{cmp}(t) = \tilde{C}_i \tilde{x}(t) \tag{4.21}$$

where,

$$\tilde{x}(t) = \begin{bmatrix} x(t)^T & e(t)^T \end{bmatrix}^T$$

$$\tilde{A}_i = \begin{bmatrix} A_i + B_i K_{xi} & -B_i K_{xi} & -B_i K_{fai} & 0 \\ 0 & \Theta_1 & B_{fa,i} - H_{1,i} C_i B_{fa,i} & -L_{11,i} B_{fs,i} \\ 0 & \Theta_2 & -H_{2,i} C_i B_{fa,i} & -L_{12,i} D_{fs,i} \\ 0 & \Theta_3 & -H_{3,i} C_i B_{fa,i} & -L_{12,i} D_{fs,i} \end{bmatrix}$$

$$\tilde{B}_i = \begin{bmatrix} B_{d,i} & 0 & 0 \\ B_{d,i} - H_{1,i} C_i D_i & 0 & -H_{1,i} D_{fs,i} \\ -H_{2,i} C_i D_i & I_{q1} & -H_{2,i} D_{fs,i} \\ -H_{3,i} C_i D_i & 0 & I_{q2} - H_{3,i} D_{fs,i} \end{bmatrix}, \tilde{C}_i = \begin{bmatrix} C_{x,i} & C_{ex,i} & C_{efa,i} & C_{efs,i} \end{bmatrix}$$

and

$$H_i = \begin{bmatrix} H_{1,i} \\ H_{2,i} \\ H_{3,i} \end{bmatrix}, L_{1,i} = \begin{bmatrix} L_{11,i} \\ L_{12,i} \\ L_{13,i} \end{bmatrix}, \Theta_1 = A_i - H_{1,i} C_i A_i - L_{11,i} C_i$$

$$\Theta_2 = -H_{2,i} C_i A_i - L_{12,i} C_i, \Theta_3 = -H_{3,i} C_i A_i - L_{13,i} C_i$$

Fault Tolerance during Unmatched period

During the unmatched period, jth subsystem and ith observer are switched together, see Fig.2.7. To formulate the problem into H_∞ filtering for $y_{cmp}(t)$, during unmatched period, from 4.10 and 4.20 we augment the switched system and UIO, into the following compact representation

$$\dot{\tilde{x}}(t) = \tilde{A}_{ij}\tilde{x}(t) + \tilde{B}_{ij}d(t)$$
$$y_{cmp}(t) = \tilde{C}_{ij}\tilde{x}(t) \tag{4.22}$$

where,

$$\tilde{x}(t) = \begin{bmatrix} x(t)^T & e(t)^T \end{bmatrix}^T$$

$$\tilde{A}_{ij} = \begin{bmatrix} A_j + B_jK_{x,j} & -B_jK_{x,j} & -B_jK_{fa,j} & 0 \\ 0 & \Phi_1 & B_{fa,j} - H_{1,i}C_jB_{fa,j} & -L_{11,j}B_{fs,j} \\ 0 & \Phi_2 & -H_{2,j}C_jB_{fa,j} & -L_{12,j}D_{fs,j} \\ 0 & \Phi_3 & -H_{3,j}C_jB_{fa,j} & -L_{12,j}D_{fs,j} \end{bmatrix}$$

$$\tilde{B}_{ij} = \begin{bmatrix} B_{d,j} & 0 & 0 \\ B_{d,j} - H_{1,i}C_jD_j & 0 & -H_{1,i}D_{fs,j} \\ -H_{2,i}C_jD_j & I_{q1} & -H_{2,i}D_{fs,j} \\ -H_{3,i}C_jD_j & 0 & I_{q2} - H_{3,i}D_{fs,j} \end{bmatrix}, \tilde{C}_{ij} = \begin{bmatrix} C_{x,j} & C_{ex,j} & C_{efa,j} & C_{efs,j} \end{bmatrix}$$

and

$$H_i = \begin{bmatrix} H_{1,i} \\ H_{2,i} \\ H_{3,i} \end{bmatrix}, L_{1,i} = \begin{bmatrix} L_{11,i} \\ L_{12,i} \\ L_{13,i} \end{bmatrix}, \Phi_1 = A_j - H_{1,i}C_jA_j - L_{11,i}C_j$$

$$\Phi_2 = -H_{2,i}C_jA_j - L_{12,i}C_j, \Phi_3 = -H_{3,i}C_jA_j - L_{13,i}C_j$$

4.4.1 Design Strategy

Complete proposed strategy in block diagram form is shown in 4.1. Following theorem presents main results of proposed work.

4.4.2 Main Results

Theorem 4. *Suppose the residual generator (4.21) and (4.22) satisfy the assumptions A1-A3, A5, if, there exist symmetric positive definite matrix* $P_i > 0, P_{ij} > 0$*;* $i \neq j; i, j \in N$ *then for a given scalar* $\alpha_i \geq 0, \rho_i \geq 0, \mu_1 \geq 1, \mu_2 \geq 1, \beta_i \geq 1$*, while any switching*

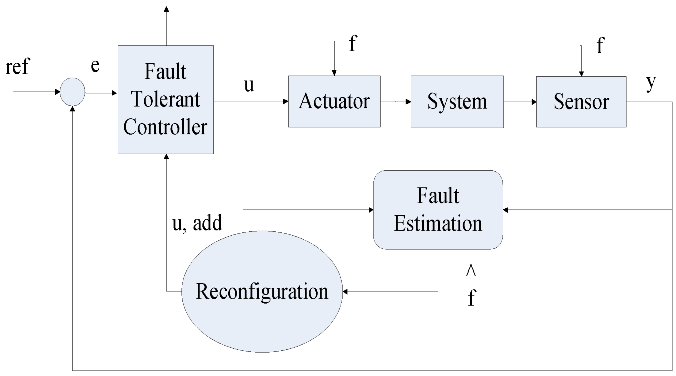

Figure 4.1: Proposed strategy: FE and FTC

signal with ADT, $\tau_\alpha > \tau_\alpha{}^* = \frac{ln(\mu 1\mu 2)}{\zeta^*}, 0 < \zeta^* < \alpha,$

$$\int_0^\infty (y_{cmp}^T)(y_{cmp})dt < \gamma_i^2 \int_0^\infty (d^T)(d)dt \tag{4.23}$$

and

$$\int_0^\infty (y_{cmp}^T)(y_{cmp})dt < \gamma_{ij}^2 \int_0^\infty (d^T)(d)dt \tag{4.24}$$

such that the following set of LMIs has a solution;

$$\begin{bmatrix} P_{11ij} & P_{12ij} \\ * & P_{22ij} \end{bmatrix} > 0 \quad , \quad \begin{bmatrix} P_{11i} & P_{12i} \\ * & P_{22i} \end{bmatrix} > 0 \tag{4.25}$$

$$\begin{bmatrix} P_{11j} & P_{12j} \\ * & P_{22j} \end{bmatrix} \leq \mu_1 \begin{bmatrix} P_{11ij} & P_{12ij} \\ * & P_{22ij} \end{bmatrix} \tag{4.26}$$

$$\begin{bmatrix} P_{11ij} & P_{12ij} \\ * & P_{22ij} \end{bmatrix} \leq \mu_2 \begin{bmatrix} P_{11i} & P_{12i} \\ * & P_{22i} \end{bmatrix} \tag{4.27}$$

$$\begin{bmatrix} \Psi_{11} & \Psi_{12} & P_{11i}B_i + P_{12i}B_i & \Psi_{14} & \Psi_{15} & \Psi_{16} \\ * & \Psi_{22} & P_{12i}{}^T B_i + P_{22i}B_i & \Psi_{24} & \Psi_{25} & \Psi_{26} \\ * & * & -\gamma_i{}^2 I & 0 & 0 & 0 \\ * & * & * & -\gamma_i{}^2 I & 0 & \Psi_{46} \\ * & * & * & * & -\gamma_i{}^2 I & \Psi_{56} \\ * & * & * & * & * & -I \end{bmatrix} < 0 \tag{4.28}$$

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & \Omega_{15} & C_j^T H_i^T \\ * & \Omega_{22} & \Omega_{23} & \Omega_{24} & \Omega_{25} & -C_i^T H_i^T \\ * & * & -\gamma_{ij}{}^2 I & 0 & 0 & \Omega_{36} \\ * & * & * & -\gamma_{ij}{}^2 I & 0 & D_{dj}^T H_i^T \\ * & * & * & 0 & -\gamma_{ij}{}^2 I & \Omega_{56} \\ * & * & * & * & * & -I \end{bmatrix} < 0 \tag{4.29}$$

where,

$$\Psi_{11} = A_i{}^T P_{11i} + P_{11i} A_i + \alpha_i P_{11i} - C_i^T L_i^T P_{12i}^T - P_{12i} L_i C_i$$

$$\Psi_{12} = A_i{}^T P_{12i} + P_{12i} A_i + \alpha_i P_{12i} + P_{12i} L_i C_i - C_i^T L_i^T P_{22i},$$

$$\Psi_{15} = P_{11i} B_{fi} - P_{12i} L_i D_{fi}$$

$$\Psi_{14} = P_{11i} B_{di} - P_{12i} L_i D_{di}, \Psi_{16} = C_i^T H_i^T$$

$$\Psi_{22} = A_i{}^T P_{22i} + \alpha_i P_{22i} + C_i{}^T L_i^T P_{22i} + P_{22i} A_i, + P_{22i} L_i C_i,$$

$$\Psi_{26} = -C_i^T H_i^T, \Psi_{56} = D_{fi}^T H_i^T - I$$

$$\Psi_{24} = P_{12i}{}^T B_{di} - P_{22i} L_i D_{di}, \Psi_{46} = -D_{di}^T H_i^T,\ \Psi_{25} = P_{12i} B_{fi} - P_{22i} L_i D_{fi},$$

$$\Psi_{45} = D_{di}{}^T D_{fi}{}^{-T} \beta_i{}^T$$

$$\Omega_{11} = A_j{}^T P_{11ij} + P_{11ij} A_j - C_j^T L_i^T P_{12ij}^T - \rho_i P_{11ij} - P_{12ij} L_i C_j,$$

$$\Omega_{15} = P_{11ij} B_{fj} - P_{12ij} L_i D_{fj}$$

$$\Omega_{12} = A_j{}^T P_{12ij} + P_{12ij} A_i - \rho_i P_{12ij} + P_{12ij} L_i C_i - C_j^T L_i^T P_{22ij}$$

$$\Omega_{13} = P_{11ij} B_j + P_{12ij} B_i - P_{12ij} L_i D_j + P_{12ij} L_i D_i$$

$$\Omega_{14} = P_{11ij} B_{dj} - P_{12ij} L_i D_{dj}$$

$$\Omega_{22} = A_j{}^T P_{22ij} - \rho_i P_{22ij} + C_i{}^T L_i^T P_{22ij} + P_{22ij} A_i + P_{22ij} L_i C_i$$

$$\Omega_{23} = P_{ij}^T B_j + P_{22ij} B_i - P_{22ij} L_i D_j + P_{22ij} L_i D_i,$$

$$\Omega_{24} = P_{12ij}{}^T B_{dj} - P_{22ij} L_i D_{dj}$$

$$\Omega_{25} = P_{12ij}{}^T B_{fj} - P_{22ij} L_i D_{fj}$$

$$\Omega_{36} = D_j^T H_i^T - D_i^T H_i^T$$

$$\Omega_{56} = D_{fj}^T H_i^T - I$$

Proof. For the desired L_i and H_i our strategy is to find out H_∞ norm of $G_{ed}, \forall i \in \{1, 2, ...N\}$ while considering matched and unmatched periods as follows.

Matched Period

We consider augmented system (4.21), in this duration. Using Lemma 3 (Appendix),

$$\dot{V}_i(\tilde{x}(t)) \leq -\alpha V_i(\tilde{x}(t)) - r^T r(t) + \gamma_i^2 \omega(t)^T \omega(t) \tag{4.30}$$

Considering the following Lyapunov function,

$$V_i(\tilde{x}(t)) = \tilde{x}^T(t) P_i \tilde{x}(t) \tag{4.31}$$

Differentiating (4.31) along the trajectory of (4.21)

$$\dot{V}_i(\tilde{x}(t)) = \dot{\tilde{x}}^T(t) P_i \tilde{x}(t) + \tilde{x}^T(t) P_i \dot{\tilde{x}}(t) \tag{4.32}$$

after substituting (4.31), (4.32), and $y_{cmp}(t)$ from (4.21) in (4.30),

$$\begin{aligned} &\dot{\tilde{x}}^T(t) P_i \tilde{x}(t) + \tilde{x}^T(t) P_i \dot{\tilde{x}}(t) + \left[\tilde{C}_i \tilde{x}(t) + \tilde{D}_i \omega(t)\right]^T \\ &\left[\tilde{C}_i \tilde{x}(t) + \tilde{D}_i \omega(t)\right] \leq -\alpha \tilde{x}^T(t) P_i \tilde{x}(t) + \gamma_i^2 \omega(t)^T \omega(t). \end{aligned} \tag{4.33}$$

Substituting the expression for $\dot{\tilde{x}}(t)$ from (4.21) in (4.33),

$$\begin{aligned} &\left[\tilde{A}_i \tilde{x}(t) + \tilde{B}_i \omega(t)\right]^T P_i \tilde{x}(t) + \tilde{x}^T(t) P_i \left[\tilde{A}_i \tilde{x}(t) + \tilde{B}_i \omega(t)\right] \\ &+ (\tilde{x}^T(t) \tilde{C}_i^T + \omega^T(t) \tilde{D}_i^T)(\tilde{C}_i \tilde{x}(t) + \tilde{D}_i \omega(t)) \\ &\leq -\alpha \tilde{x}^T(t) P_i \tilde{x}(t) + \gamma_i^2 \omega(t)^T \omega(t)) \end{aligned} \tag{4.34}$$

(4.34) can be written easily in following form

$$\begin{aligned} &\left[\tilde{x}^T(t) \tilde{A}_i^T P_i + \omega^T(t) \tilde{B}_i^T P_i\right] \tilde{x}(t) \\ &+ \tilde{x}^T(t) P_i \tilde{A}_i \tilde{x}(t) + \tilde{x}^T(t) P_i \tilde{B}_i \omega(t) + \tilde{x}^T(t) \tilde{C}_i^T \tilde{C}_i \tilde{x}(t) \\ &+ \tilde{x}^T(t) \tilde{C}_i^T \tilde{D}_i \omega(t) + \omega^T(t) \tilde{D}_i^T \tilde{C}_i \tilde{x}(t) \\ &+ \omega^T(t) \tilde{D}_i^T \tilde{D}_i \omega(t)) + \alpha \tilde{x}^T(t) P_i \tilde{x}(t) \\ &- {\gamma_i}^2 \omega^T(t) \omega(t) \leq 0 \end{aligned} \tag{4.35}$$

Further, the above inequality can be written as

$$\begin{bmatrix} \tilde{x}^T(t) & \omega^T(t) \end{bmatrix} M \begin{bmatrix} \tilde{x}(t) \\ \omega(t) \end{bmatrix} \leq 0 \quad where, \tag{4.36}$$

$$M = \begin{bmatrix} \tilde{A}_i^T P_i + P_i \tilde{A}_i + \tilde{C}_i^T \tilde{C}_i + \alpha P_i & P_i \tilde{B}_i + \tilde{C}_i^T \tilde{D}_i \\ * & \tilde{D}_i^T \tilde{D}_i - \gamma_i^2 I \end{bmatrix}$$

for (4.36) to hold, it is required that

$$M < 0 \tag{4.37}$$

After Schur's compliment is applied to (4.37), we get

$$\begin{bmatrix} \tilde{A}_i^T P_i + P_i \tilde{A}_i + \alpha P_i & P_i \tilde{B}_i & \tilde{C}_i^T \\ * & -\gamma_i^2 I & \tilde{D}_i^T \\ * & * & -I \end{bmatrix} < 0 \tag{4.38}$$

which is the basic form of LMI (4.28). Notice that, LMIs (4.25)-(4.27) are general requirements for model-based FD in asynchronous switching paradigm [93].

Unmatched period

We consider augmented system (4.22), in this duration,

$$\dot{V_{ij}}(\tilde{x}(t)) \leq \rho V_{ij}(\tilde{x}(t)) - r(t)^T r(t) + \gamma_{ij}^2 \omega(t)^T \omega(t) \tag{4.39}$$

where $i \neq j$ and $i, j \in N$. During unmatched period,

$$V_{ij}(\tilde{x}(t)) = \tilde{x}^T(t) P_{ij} \tilde{x}(t) \tag{4.40}$$

To derive further the results of Theorem 1, we apply the same procedure as it is done earlier for matched period. Thus, LMI (4.29) of Theorem 4 is derived. □

Remark 3. *The formulation of the fault estimation problem formally sounds reasonable for an LMI (linear matrix inequality) solution, but the estimation performance can be poor if no further constraint on the faults is assumed. In general, the augmented form of* *the system model can only work well, if the faults are almost constant in a time interval. From the observer view point, it is the design of a Pl-Observer. It is well-known that such an observer can work well only if the change of the estimated variables(in a time interval) is (strongly) bounded.*

4.5 Case Study

In this section simulation results for a case study of numerical example are presented. Subsystem 1 is activated when $\sigma(t) = 1$ and subsystem 2 when $\sigma(t) = 0$. Based on our proposed FTC approach we are able to tolerate actuator and sensor. Details of temporal switching behaviour of subsystems, observers and faults can be seen in Fig. 4.2. Switching signal $\sigma(t)$ is applied according to ADT value of 1.6894 for parameters $\mu_1 = 1.5, \mu_2 = 1.5, \alpha = 0.5, \zeta^* = 0.48$ according to ADT definition, in Chapter 2. Further, we take the disturbance signal of L_2 norm bounded by $\delta_{d,2} \leq 1$ for each mode. The considered switched dynamical system consisting of two modes is as follows

$$A_1 = \begin{bmatrix} -1 & 1 \\ 2 & -1 \end{bmatrix}, B_1 = \begin{bmatrix} 0.5 & 1 \\ 1 & 2 \end{bmatrix} C_1 = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}, B_{d1} = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, B_{fa,1} = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -1 & 1 \\ 3 & 1 \end{bmatrix}, B_2 = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}, C_2 = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}, B_{d2} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, B_{fa,2} = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix},$$

$$D_{fs,1} = \begin{bmatrix} 0.1 \\ 0.5 \end{bmatrix}, D_{fs,2} = \begin{bmatrix} 0.1 \\ 0.5 \end{bmatrix},$$

Solving LMIs (4.25)-(4.29) , Observer parameters are found to be:

$$M_1 = \begin{bmatrix} -23.849 & 7.377 & -2.003 & -0.878 \\ -11.067 & -30.772 & 2.521 & -10.610 \\ 37.019 & 27.341 & -18.725 & -13.933 \\ -25.205 & -51.807 & 3.877 & -18.757 \end{bmatrix}, H_1 = \begin{bmatrix} -3.507 & 4.007 \\ 4.554 & -4.054 \\ -37.450 & 37.450 \\ 7.754 & -7.754 \end{bmatrix}$$

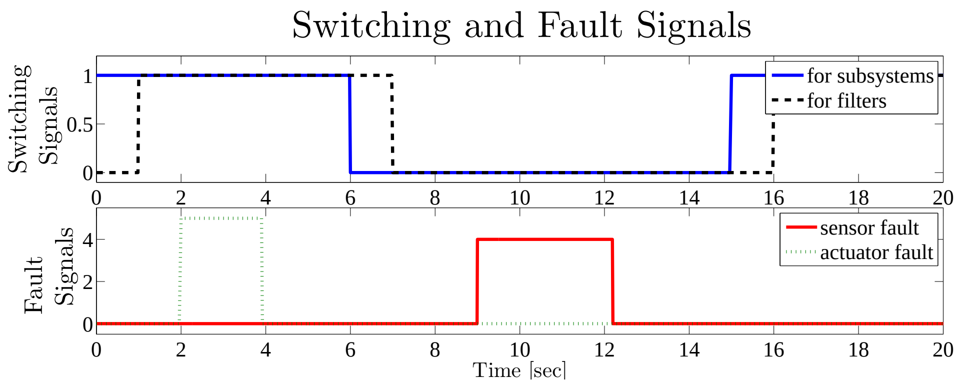

Figure 4.2: Switching signals: for subsystems $\sigma(t)$; and for observers $\sigma'(t)$: Fault signals: of actuator fault $f_1(t)$; and of sensor fault $f_2(t)$

$$G_1 = \begin{bmatrix} 4.007 & -4.007 & 0 & -1.652 \\ -5.054 & 5.054 & 0 & 1.571 \\ 37.450 & -37.450 & 1.000 & -14.980 \\ -7.754 & 7.754 & 0 & 4.1018 \end{bmatrix}, L_1 = \begin{bmatrix} 191.071 & -193.074 \\ -275.463 & 277.990 \\ 489.433 & -508.158 \\ -432.113 & 435.990 \end{bmatrix},$$

$$M_2 = \begin{bmatrix} -27.080 & 6.641 & -1.877 & -0.993 \\ -7.908 & -30.392 & 2.383 & -10.734 \\ -9.612 & 28.263 & -19.162 & -14.589 \\ -13.145 & -51.357 & 4.005 & -18.159 \end{bmatrix}, H_2 = \begin{bmatrix} -3.254 & 3.755 \\ 4.267 & -3.767 \\ -38.324 & 38.324 \\ 8.011 & -8.011 \end{bmatrix}$$

$$G_2 = \begin{bmatrix} 3.754 & -3.755 & 0 & -1.552 \\ -4.766 & 4.767 & 0 & 1.456 \\ 38.324 & -38.324 & 1.000 & -15.330 \\ -8.011 & 8.011 & 0 & 4.204 \end{bmatrix}, L_2 = \begin{bmatrix} 186.078 & -189.836 \\ -278.249 & 283.012 \\ 673.380 & -711.704 \\ -470.462 & 478.466 \end{bmatrix},$$

and $\gamma_1 = 0.02, \gamma_2 = 0.03, \gamma_{12} = 0.05, \gamma_{21} = 0.06$. Then, the switched system is simulated according to switching instants and faults occurrences shown in Fig. 4.2. From figures 4.3 to 4.4, case of actuator fault is studied. Actuator fault occurs is assumed to be occurred at 2 sec. 4.3 shows the actuator fault, its estimate by our proposed FE UIO and estimation error. Normal output, faulty output, and compensated output by employing our proposed FTC strategy is depicted in Fig. 4.4. The Figure shows the compensated output, in dotted line, nearly close to normal output.

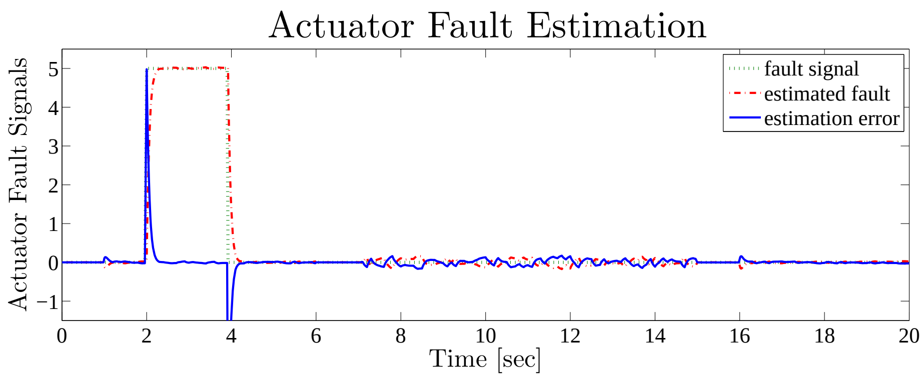

Figure 4.3: Actuator fault signal, estimated actuator fault and estimation error

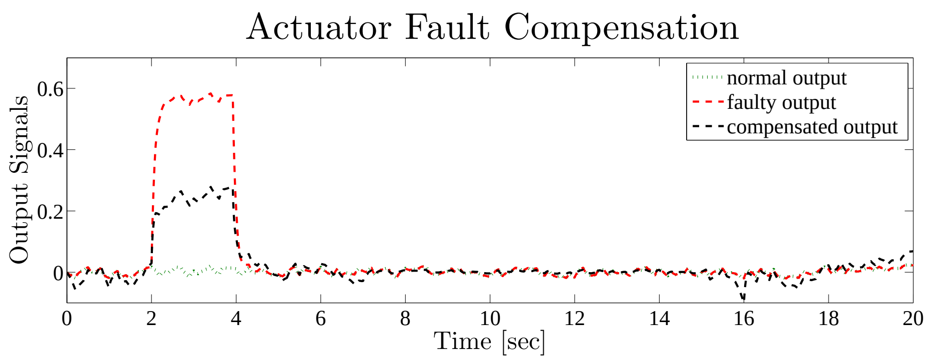

Figure 4.4: Normal output, faulty output, and compensated output in case of actuator fault

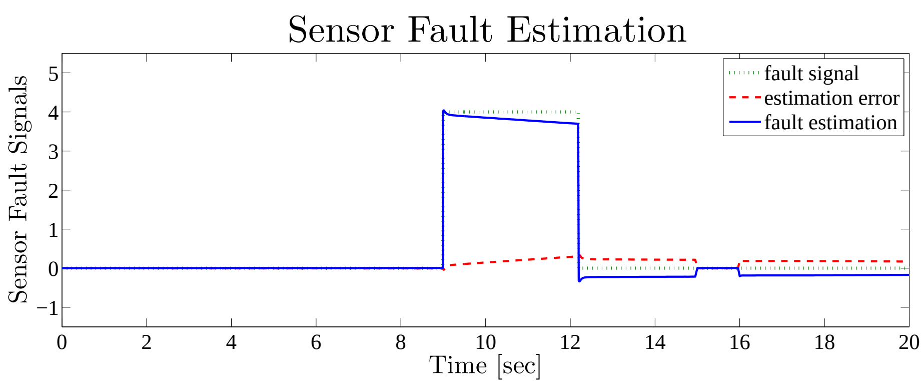

Figure 4.5: Sensor fault signal, estimated actuator fault and estimation error

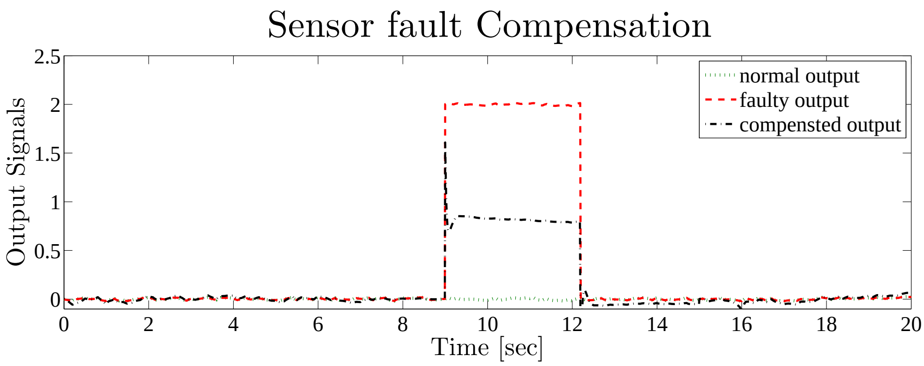

Figure 4.6: Normal output, faulty output, and compensated output in case of sensor fault

Similarly, for the second case of sensor fault, Fig. 4.5 shows the sensor fault, its estimate by our proposed FE UIO, and estimation error. Normal output, faulty output, and compensated output by employing our proposed FTC strategy is depicted in Fig. 4.6. These results show the successfully compensated actuator and sensor fault.

4.6 Summary

In this chapter, the problem of faults estimation and fault tolerance for switched systems has been addressed. To estimate actuator and sensor faults H_∞ UIO is strategy is proposes. To solve the FE problem, switching between observers and subsystems is considered to be of asynchronous nature. After this step, fault tolerance for both types of faults is achieved by employing a reconfiguration approach of active FTC. Further, in this work ADT switching constraint is assumed and results are derived in the form of LMIs. To show the effectiveness of proposed approach, case study of a switched system is worked out. Successful results of fault compensations are obtained for both cases of fault.

Chapter 5

Fault Detection in Switched Time-Delay Systems

In this chapter, fault detection for switched time-delay systems under asynchronous switching is discussed. Fault detection problem is formulated into H_∞ filtering problem and sufficient conditions are proposed, in terms of linear matrix inequalities using Lyapunov-Krasovskii functional.

5.1 Introduction

This chapter addresses the fault detection problem for a class of continuous time switched time-delay systems. Time-delay is considered in states, while average dwell time switching is considered. Fault detection filter (FDF) is designed to generate residual signal. Switching signal of FDF is delayed with respect to that of subsystems. In this way, asynchronous switching arises between switched system and fault detection filter. The fault detection problem is formulated into H_∞ filtering problem using Lyapunov-Krasovskii functional, during matched and mismatched periods of asynchronous switching. Sufficient conditions for solving the problem are established in terms of linear matrix inequalities. At the end, proposed results are simulated on an example to show the effectiveness of the proposed method.

The rest of the chapter is organized as follows: Next, research contribution of this chapter is presented. In Section 5.2 problem is formulated and in Section 5.3 solution is proposed. In Section 5.4, threshold is computed for the case study and residual signals are evaluated. In Section 5.5, application results are demonstrated to show the effectiveness of the approach, followed by chapter summary in Section 5.6.

Research Contribution

Time-delays are one of the inherent features of many engineering systems, for example, electronics, communication, hydraulic, and chemical systems. Presence of time-delays may cause, poor performance, oscillation. or instability. Hence, it is important to study switched systems with presence of time-delays. It becomes a challenging problem when dealing the switchings with time-delays. There are few results in literature on the topic, see for instance, [109, 110, 111, 112, 113, 69, 70, 89, 114].

Survey on fault detection for switched time-delay systems reveals that there are very few results available in the literature. In [69], conditions for existence of fault detection filter for discrete-time switched time-delay systems, under an arbitrary switching signal, are obtained using switched Lyapunov functional approach. Then, in [70], problem of robust fault detection filter for continuous-time switched systems with state delays was discussed. Lyapunov-Krasovskii functional method was used. Delay-dependent sufficient condition for the solvability of fault detection problem is investigated, for a class of discrete-time switched linear systems with time-varying delays, in [115]. A robust fault detection filter that guarantees both sensitivity to faults and robustness to disturbances for discretetime switched systems with state delays has been investigated in [116].

In this chapter, the fault detection problem is studied when there are state delays in switched system. In addition, filters are switching asynchronously with switched time-delay systems. Since the state delay is of continuous nature, while switching delay of filters is of discrete nature, the emergence of these two types of delays make the problem of fault detection more difficult. Few results on fault detection for switched time-delay system under synchronous switching are available in literature [69, 74, 71]. Here, to study the asynchronous switching case, fault detection problem is transformed to H_∞ filtering using piece-wise Lyapunov-Krasovskii functional method. Sufficient conditions are derived in the form of linear matrix inequalities (LMIs). Filter parameters are designed by solving derived LMIs.

5.2 Problem Formulation: Fault Detection in Switched Time-Delay Systems

System description

Consider the following class of continuous-time switched time-delay systems

$$\begin{aligned}
\dot{x}(t) &= A_{\sigma(t)}x(t) + A_{h\sigma(t)}x(t-h) + B_{\sigma(t)}u(t) + B_{d\sigma(t)}d(t) + B_{f\sigma(t)}f(t) \\
y(t) &= C_{\sigma(t)}x(t) + C_{h\sigma(t)}x(t-h) + D_{\sigma(t)}u(t) + D_{d\sigma(t)}d(t) + D_{f\sigma(t)}f(t) \\
x(\theta) &= \phi(\theta), \theta \in [-h, 0]
\end{aligned} \tag{5.1}$$

where $x(t) \in R^n$ is a state vector, $u(t) \in R^r$ is a control input vector, $y(t) \in R^m$ is an output vector, $d(t) \in R^p$ is an unknown input (disturbances, noise) vector, $f(t) \in R^q$ is a fault vector, $\phi(\theta)$ is a continuous initial function on $[-h, 0]$, $\sigma(t)$ is a switching signal which is a piecewise constant function $\sigma : [0, \infty) \longrightarrow \rho$. Such a function σ has a finite number of switching times and takes a constant value on every interval between two consecutive switching times. The role of $\sigma(t)$ is to specify, at each instant t, the index $\sigma(t) \in \rho$ of the active subsystem. Also, $A_{\sigma(t)}, A_{h\sigma(t)}, B_{\sigma(t)}, C_{\sigma(t)}, C_{h\sigma(t)}, D_{\sigma(t)}, B_{d\sigma(t)}, D_{d\sigma(t)}, B_{f\sigma(t)}, D_{f\sigma(t)}$ are the systems, disturbances and fault coupling matrices with appropriate dimensions.

Problem 5. *Given the switched time-delay system (5.1) subject to actuator and sensor faults, under the effect of disturbances and noise, design a fault detection filter (FDF) such that the system satisfying the average dwell time (ADT) under asynchronous switching is exponentially stable with H_∞ performance, $\| G_{r\omega} \|_\infty < \gamma$.*

5.3 Solution to the Fault detection Problem

5.3.1 H_∞ Fault Detection

In order to detect the actuator and sensor faults residual $r(t)$ is generated. To achieve the objective H_∞ fault detection filter (FDF) is designed such that effects of control input $u(t)$, and disturbances $d(t)$ on $r(t)$ is minimized. To this end, the following switched

FDF model is used.

$$\begin{aligned}\dot{\hat{x}}(t) &= A_{\sigma'(t)}\hat{x}(t) + A_{h\sigma'(t)}\hat{x}(t-h) + B_{\sigma'(t)}u(t) - L_{\sigma'(t)}(y(t) - \hat{y}(t))\\ \hat{y}(t) &= C_{\sigma'(t)}\hat{x}(t) + C_{h\sigma'(t)}\hat{x}(t-h) + D_{\sigma'(t)}u(t)\\ r(t) &= H_{\sigma'(t)}(y(t) - \hat{y}(t))\end{aligned} \tag{5.2}$$

Where $L_{\sigma'(t)} \in R^{n\times m}$ and $H_{\sigma'(t)} \in R^{q\times m}$ are the parameters of the filter to be designed with respect to each subsystem $i \in \{1, 2, ...N\}$, $r(t)$ is the residual signal, which is difference between actual output $y(t)$ and estimated output $\hat{y}(t)$. Similar to the system, the switching between different modes of the filter depends on the switching signal $\sigma'(t)$, shown in Fig. 2.6. Practically, there exists the phenomenon of asynchronous switching between the filter and the system in most of the cases. The asynchronous switching problem is studied in detail in Chapter 2. Due to asynchronous switching, problem is studied during matched and unmatched periods as follows.

Matched period

During the matched period, *ith* subsystem and *ith* filter are in operation, see Fig.2.7. To formulate the problem into H_∞ filtering for $r(t)$, we augment the switched system (5.1) and detection filter (5.2) into the following compact representation during matched period

$$\begin{aligned}\dot{\tilde{x}}(t) &= \tilde{A}_i\tilde{x}(t) + \tilde{A_{hi}}\tilde{x}(t-h) + \tilde{E}_i f(t) + \tilde{B}_i\tilde{\omega}(t)\\ r(t) &= \tilde{C}_i\tilde{x}(t) + \tilde{C_{hi}}\tilde{x}(t-h) + \tilde{F}_i f(t) + \tilde{D}_i\tilde{\omega}(t),\end{aligned} \tag{5.3}$$

where,

$$\begin{aligned}\tilde{x}(t) &= \begin{bmatrix} x(t)^T & \hat{x}(t)^T \end{bmatrix}^T, \tilde{x}(t-h) = \begin{bmatrix} x(t-h)^T & \hat{x}(t-h)^T \end{bmatrix}^T,\\ \tilde{\omega}(t) &= \begin{bmatrix} u(t)^T & d(t)^T \end{bmatrix}^T, \tilde{A}_i = \begin{bmatrix} A_i & 0 - L_iC_iA_i + L_iC_i \end{bmatrix}, \tilde{D}_i = \begin{bmatrix} 0 & H_iD_{di} \end{bmatrix}\\ \tilde{A}_{hi} &= \begin{bmatrix} A_{hi} & 0 \\ -L_iC_{hi} & A_{hi} + L_iC_{hi} \end{bmatrix}, \tilde{E}_i = \begin{bmatrix} B_{fi} \\ -L_iD_{fi} \end{bmatrix}, \tilde{B}_i = \begin{bmatrix} B_i & B_{di} \\ B_i & -L_iD_{di} \end{bmatrix},\\ \tilde{C}_i &= \begin{bmatrix} H_iC_i & -H_iC_i \end{bmatrix}, \tilde{C}_{hi} = \begin{bmatrix} H_iC_{hi} & -H_iC_{hi} \end{bmatrix}, \tilde{F}_i = \begin{bmatrix} H_iD_{fi} \end{bmatrix}\end{aligned}$$

Unmatched period

During the unmatched period, jth subsystem and ith filter are switched together, see Fig.2.7. We augment the switched system (5.1) and detection filter (5.2) into the following compact representation during unmatched period

$$\begin{aligned}\dot{\tilde{x}}(t) &= \tilde{A}_{ij}\tilde{x}(t) + \tilde{A}_{hij}\tilde{x}(t-h) + \tilde{E}_{ij}f(t) + \tilde{B}_{ij}\tilde{\omega}(t)\\ r(t) &= \tilde{C}_{ij}\tilde{x}(t) + \tilde{C}_{hij}\tilde{x}(t-h) + \tilde{F}_{ij}f(t) + \tilde{D}_{ij}\tilde{\omega}(t),\end{aligned} \tag{5.4}$$

where,

$$\tilde{x}(t) = \begin{bmatrix} x(t)^T & \hat{x}(t)^T \end{bmatrix}^T, \tilde{x}(t-h) = \begin{bmatrix} x(t-h)^T & \hat{x}(t-h)^T \end{bmatrix}^T,$$

$$\tilde{\omega}(t) = \begin{bmatrix} u(t)^T d(t)^T \end{bmatrix}^T, \tilde{A}_{ij} = \begin{bmatrix} A_j & 0 \\ -L_iC_j & L_iC_i \end{bmatrix}, \tilde{A}_{hij} = \begin{bmatrix} A_{hj} & 0 \\ -L_iC_{hj} & L_iC_{hi} \end{bmatrix}$$

$$\tilde{E}_{ij} = \begin{bmatrix} B_{fj} \\ -L_iD_{fj} \end{bmatrix}, \tilde{B}_{ij} = \begin{bmatrix} B_j & B_{dj} \\ -L_iD_j + L_iD_i & -L_iD_{dj} \end{bmatrix}, \tilde{C}_{ij} = \begin{bmatrix} H_iC_j & -H_iC_i \end{bmatrix}$$

$$\tilde{C}_{hij} = \begin{bmatrix} H_iC_{hj} & -H_iC_{hi} \end{bmatrix}, \tilde{F}_{ij} = \begin{bmatrix} H_iD_{fj} \end{bmatrix}, \tilde{D}_{ij} = \begin{bmatrix} H_iD_j - H_iD_i & H_iD_{dj} \end{bmatrix}$$

Following theorem presents main results of proposed work.

5.3.2 Main Results

Theorem 5. *Suppose the residual generator (5.3) and (5.4) satisfy the assumptions A1-A3, if, there exist* $P_i \geq 0, P_{ij} \geq 0, Q_i \geq 0, Q_{ij} \geq 0$; $i \neq j$, *and* $i, j \in N$, *then for a given scalar* $\lambda_m \geq 0, \lambda_u \geq 0, \mu_1 \geq 1, \mu_2 \geq 1, \beta_i \geq 1$, *while any switching signal with ADT,* $\tau_\alpha > {\tau_\alpha}^* = \frac{ln(\mu 1\mu 2)}{\zeta^*}, 0 < \zeta^* < \alpha$,

$$\int_0^\infty (r^T)(r)dt < \gamma_i^2 \int_0^\infty (\omega^T)(\omega)dt \tag{5.5}$$

and

$$\int_0^\infty (r^T)(r)dt < \gamma_{ij}^2 \int_0^\infty (\omega^T)(\omega)dt \tag{5.6}$$

such that the following set of LMIs has a solution;

$$\begin{bmatrix} P_{11j} & P_{12j} \\ * & P_{22j} \end{bmatrix} \leq \mu_1 \begin{bmatrix} P_{11ij} & P_{12ij} \\ * & P_{22ij} \end{bmatrix} \tag{5.7}$$

$$\begin{bmatrix} P_{11ij} & P_{12ij} \\ * & P_{22ij} \end{bmatrix} \leq \mu_2 \begin{bmatrix} P_{11i} & P_{12i} \\ * & P_{22i} \end{bmatrix} \tag{5.8}$$

$$\begin{bmatrix} Q_{11j} & Q_{12j} \\ * & Q_{22j} \end{bmatrix} \leq \mu_1 \begin{bmatrix} Q_{11ij} & Q_{12ij} \\ * & Q_{22ij} \end{bmatrix} \tag{5.9}$$

$$\begin{bmatrix} Q_{11ij} & Q_{12ij} \\ * & Q_{22ij} \end{bmatrix} \leq \mu_2 \begin{bmatrix} Q_{11i} & Q_{12i} \\ * & Q_{22i} \end{bmatrix} \tag{5.10}$$

$$\begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} & \Psi_{14} & \Psi_{15} & \Psi_{16} & \Psi_{17} & \Psi_{18} \\ * & \Psi_{22} & \Psi_{23} & \Psi_{24} & \Psi_{25} & \Psi_{26} & \Psi_{27} & \Psi_{28} \\ * & * & \Psi_{33} & \Psi_{34} & \Psi_{35} & \Psi_{36} & \Psi_{37} & \Psi_{38} \\ * & * & * & \Psi_{44} & \Psi_{45} & \Psi_{46} & \Psi_{47} & \Psi_{48} \\ * & * & * & * & \Psi_{55} & \Psi_{56} & \Psi_{57} & \Psi_{58} \\ * & * & * & * & * & \Psi_{66} & \Psi_{67} & \Psi_{68} \\ * & * & * & * & * & * & \Psi_{77} & \Psi_{78} \\ * & * & * & * & * & * & * & \Psi_{88} \end{bmatrix} < 0 \tag{5.11}$$

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & \Omega_{15} & \Omega_{16} & \Omega_{17} & \Omega_{18} \\ * & \Omega_{22} & \Omega_{23} & \Omega_{24} & \Omega_{25} & \Omega_{26} & \Omega_{27} & \Omega_{28} \\ * & * & \Omega_{33} & \Omega_{34} & \Omega_{35} & \Omega_{36} & \Omega_{37} & \Omega_{38} \\ * & * & * & \Omega_{44} & \Omega_{45} & \Omega_{46} & \Omega_{47} & \Omega_{48} \\ * & * & * & * & \Omega_{55} & \Omega_{56} & \Omega_{57} & \Omega_{58} \\ * & * & * & * & * & \Omega_{66} & \Omega_{67} & \Omega_{68} \\ * & * & * & * & * & * & \Omega_{77} & \Omega_{78} \\ * & * & * & * & * & * & * & \Omega_{88} \end{bmatrix} < 0 \tag{5.12}$$

$$Where, \quad \Psi_{11} = {A_i}^T P_{11i} + P_{11i} A_i + \lambda_m P_{11i} - C_i^T L_i^T P_{12i}^T - P_{12i} L_i C_i + Q_{11i}$$

$\Psi_{12} = A_i{}^T P_{12i} + P_{12i}A_i + \lambda_m P_{12i} - C_i^T L_i^T P_{22i}^T + P_{12i}L_iC_i + Q_{12i}$

$\Psi_{13} = P_{11i}A_{hi} - P_{12i}L_iC_{hi}, \Psi_{14} = P_{12i}A_{hi} + P_{12i}L_iC_{hi}$

$\Psi_{15} = P_{11i}B_{fi} - P_{12i}L_iD_{fi},\ \Psi_{16} = P_{11i}B_i + P_{12i}B_i$

$\Psi_{17} = P_{11i}B_{di} - P_{12i}L_iD_{di},\ \Psi_{18} = C_i^T H_i^T, \Psi_{25} = P_{12i}B_{fi} - P_{22i}L_iD_{fi}$

$\Psi_{22} = A_i{}^T P_{22i} + P_{22i}A_i + \lambda_m P_{22i} + C_i^T L_i^T P_{22i}^T + P_{22i}L_iC_i + Q_{22i}^T$

$\Psi_{23} = P_{12i}A_{hi} - P_{22i}B_{fi}L_iC_{hi},\ \Psi_{24} = P_{22i}A_{hi} + P_{22i}L_iC_{hi}$

$\Psi_{28} = -C_i^T H_i^T,\ \Psi_{31} = A_{hi}^T P_{11i} - C_{hi}^T L_i^T P_{12i}^T,\ \Psi_{32} = A_{hi}^T P_{12i} - C_{hi}^T L_i^T P_{22i}^T$

$\Psi_{33} = -Q_{11i}\Psi_{34} = -Q_{12i}\Psi_{35} = 0\Psi_{36} = 0\Psi_{37} = 0\Psi_{38} = C_{hi}^T H_i^T$

$\Psi_{41} = A_{hi}^T P_{12i} + C_{hi}^T L_i^T P_{12i}^T,\ \Psi_{42} = A_{hi}^T P_{22i} + C_{hi}^T L_i^T P_{22i}^T,\ \Psi_{43} = -Q_{12i}$

$\Psi_{44} = -Q_{22i},\ \Psi_{45} = 0,\ \Psi_{46} = 0,\ \Psi_{47} = 0,\ \Psi_{48} = -C_{hi}^T H_i^T$

$\Psi_{51} = B_{fi}^T P_{11i} - D_{fi}^T L_i^T P_{12i}^T, \Psi_{52} = B_{fi}^T P_{12i} - D_{fi}^T L_i^T P_{22i}^T,\ \Psi_{53} = 0, \Psi_{54} = 0$

$,\ \Psi_{55} = 0,\ \Psi_{56} = 0,\ \Psi_{57} = 0,\ \Psi_{58} = D_{fi}^T H_i^T,\ \Psi_{26} = P_{12i}B_i + P_{22i}B_i$

$\Psi_{27} = P_{12i}B_{di} - P_{22i}L_iD_{di},\ \Psi_{61} = B_i^T P_{11i} + B_i^T P_{12i},$

$\Psi_{62} = B_i^T P_{12i} + B_i^T P_{22i},\ \Psi_{63} = 0,\ \Psi_{64} =,\ \Psi_{65} = 0$

$\Psi_{66} = -\gamma_i^2 I, \Psi_{67} = 0, \Psi_{68} = 0, \Psi_{71} = B_{di}^T P_{11i} - D_{di}^T L_i^T P_{12i}^T$

$\Psi_{72} = B_{di}^T P_{12i} - D_{di}^T L_i^T P_{22i}^T, \Psi_{73} = 0, \Psi_{74} = 0, \Psi_{75} = 0, \Psi_{76} = 0$

$\Psi_{77} = -\gamma_i^2 I, \Psi_{78} = D_{di}^T H_i^T, \Psi_{81} = H_iC_i, \Psi_{82} = -H_iC_i, \Psi_{83} = H_iC_{hi}$

$\Psi_{84} = -H_iC_{hi}\Psi_{85} = H_iD_{fi}, \Psi_{86} = 0, \Psi_{87} = H_iD_{di}, \Psi_{88} = -I$

$\Omega_{11} = A_j{}^T P_{11ij} + P_{11ij}A_j - \lambda_u P_{11ij} - C_j^T L_i^T P_{12ij}^T - P_{12ij}L_iC_j + Q_{11ij}$

$\Omega_{12} = A_j{}^T P_{12ij} - \lambda_u P_{12ij} - C_j^T L_i^T P_{22ij}^T + P_{12ij}L_iC_i + Q_{12ij}$

$\Omega_{13} = P_{11ij}A_{hj} - P_{12ij}L_iC_{hj}, \Omega_{14} = P_{12ij}L_iC_{hi}$

$\Omega_{15} = P_{11ij}B_{fj} - P_{12ij}L_iD_{fj}, \Omega_{16} = P_{11ij}B_j - P_{12ij}L_iD_j + P_{12ij}L_iD_i$

$\Omega_{17} = P_{11ij}B_{dj} - P_{12ij}L_iD_{dj}, \Omega_{18} = C_j^T H_i^T$

$\Omega_{22} = -\lambda_u P_{22ij} + C_i^T L_i^T P_{22ij} + P_{22ij}L_iC_i + Q_{22ij}$

$\Omega_{23} = P_{12ij}^T A_{hj} - P_{22ij}L_iC_{hj}, \Omega_{24} = P_{22ij}L_iC_{hi}, \Omega_{25} = P_{12ij}^T B_{fj} - P_{22ij}L_iD_{fj}$

$\Omega_{26} = P_{12ij}^T B_j - P_{22i}L_iD_j + P_{22ij}L_iD_i, \Omega_{27} = P_{12ij}^T B_{dj} - P_{22ij}L_iD_{dj}$

$\Omega_{28} = -C_i^T H_i^T, \Omega_{38} = C_{hj}^T H_i^T, \Omega_{48} = -C_{hi}^T H_i^T$

$\Omega_{51} = B_{fj}^T P_{11ij}^T - D_{fj}^T L_i^T P_{12ij}^T \Omega_{52} = B_{fj}^T P_{12ij}^T - D_{fj}^T L_i^T P_{22ij}^T$

$\Omega_{53} = \Omega_{54} = \Omega_{55} = \Omega_{56} = \Omega_{63} = \Omega_{64} = \Omega_{65} = \Omega_{75} = \Omega_{73} = \Omega_{74} = \Omega_{67} = \Omega_{76} = 0$

$\Omega_{58} = D_{fj}^T H_i^T, \Omega_{61} = B_j^T P_{11ij}^T - D_j^T L_i^T P_{12ij}^T + D_i^T L_i^T P_{12ij}^T$

$\Omega_{62} = B_j^T P_{12ij}^T - D_j^T L_i^T P_{22ij}^T + D_i^T L_i^T P_{22ij}^T,\ \Omega_{71} = B_{dj}^T P_{11ij}^T - D_{dj}^T L_i^T P_{12ij}^T$

$\Omega_{72} = B_{dj}^T P_{12ij}^T - D_{dj}^T L_i^T P_{22ij}^T, \Omega_{66} = -\gamma_{ij}^2 I, \Omega_{77} = -\gamma_{ij}^2 I$

$\Omega_{68} = D_j^T H_i^T - D_i^T H_i^T, \Omega_{78} = D_{dj}^T H_i^T, \Omega_{81} = H_i C_j, \Omega_{82} = -H_i C_i,$

$\Omega_{83} = H_i C_{hj},\ \Omega_{84} = -H_i C_{hi},\ \Omega_{85} = H_i D_{fj}, \Omega_{86} = H_i D_j - H_i D_i,\ \Omega_{87} = H_i D_{dj}, \Omega_{88} = -I$

Proof. For the desired L_i and H_i our strategy is to find out H_∞ norm of $G_{ed}, \forall i \in \{1, 2, ...N\}$ while considering matched and unmatched periods as follows.

Matched Period: We consider augmented system (5.3), in this duration. Using Lemma 1,

$$\dot{V}_i(\tilde{x}(t)) \leq -\lambda_m V_i(\tilde{x}(t)) - r(t)^T r(t) + \gamma_i^2 \omega(t)^T \omega(t) \tag{5.13}$$

Considering the following Lyapunov function,

$$V_i(\tilde{x}(t)) = \tilde{x}^T(t) P_i \tilde{x}(t) + \int_{t-h}^{t} \tilde{x}^T(s) e^{\lambda m} Q_i \tilde{x}(s) d(s) \tag{5.14}$$

Differentiating (5.14) along the trajectory of (5.3)

$$\dot{V}_i(\tilde{x}(t)) = \dot{\tilde{x}}^T(t) P_i \tilde{x}(t) + \tilde{x}^T(t) P_i \dot{\tilde{x}}(t) + \tilde{x}^T(t) e^{\lambda m} Q_i \tilde{x}(t) - \tilde{x}^T(t-h) e^{\lambda m} Q_i \tilde{x}(t-h) \tag{5.15}$$

after substituting (5.14)and (5.15), in (5.13),

$$\begin{aligned} &\dot{\tilde{x}}^T(t) P_i \tilde{x}(t) + \tilde{x}^T(t) P_i \dot{\tilde{x}}(t) + \tilde{x}^T(t) e^{\lambda m} Q_i \tilde{x}(t) - \tilde{x}^T(t-h) e^{\lambda m} Q_i \tilde{x}(t-h) + \\ &\left[\tilde{C}_i \tilde{x}(t) + \tilde{C}_{hi} \tilde{x}(t-h) + \tilde{F}_i f(t) + \tilde{D}_i \tilde{\omega}(t)\right]^T \left[\tilde{C}_i \tilde{x}(t) + \tilde{C}_{hi} \tilde{x}(t-h) + \tilde{F}_i f(t) + \tilde{D}_i \tilde{\omega}(t)\right] \\ &\leq -\lambda_m \tilde{x}^T(t) P_i \tilde{x}(t) - \lambda_m \int_{t-h}^{t} \tilde{x}^T(s) e^{\lambda_m} Q_i \tilde{x}(s) d(s) + \gamma_i^2 \omega(t)^T \omega(t). \end{aligned} \tag{5.16}$$

Substituting the expression for $\dot{\tilde{x}}(t)$ from (5.3) in (5.16),

$$\left[\tilde{A}_i\tilde{x}(t) + \tilde{A}_{hi}\tilde{x}(t-h) + \tilde{E}_i f(t) + \tilde{B}_i\tilde{\omega}(t)\right]^T P_i\tilde{x}(t)$$
$$+ \tilde{x}^T(t)P_i\left[\tilde{A}_i\tilde{x}(t) + \tilde{A}_{hi}\tilde{x}(t-h) + \tilde{E}_i f(t) + \tilde{B}_i\tilde{\omega}(t)\right] + \tilde{x}^T(t)e^{\lambda m}Q_i\tilde{x}(t)$$
$$- \tilde{x}^T(t-h)e^{\lambda m}Q_i\tilde{x}(t-h)+$$
$$\left[\tilde{C}_i\tilde{x}(t) + \tilde{C}_{hi}\tilde{x}(t-h) + \tilde{F}_i f(t) + \tilde{D}_i\tilde{\omega}(t)\right]^T \left[\tilde{C}_i\tilde{x}(t) + \tilde{C}_{hi}\tilde{x}(t-h) + \tilde{F}_i f(t) + \tilde{D}_i\tilde{\omega}(t)\right]$$
$$\leq -\lambda_m\tilde{x}^T(t)P_i\tilde{x}(t) - \lambda_m\int_{t-h}^{t}\tilde{x}^T(s)e^{\lambda m}Q_i\tilde{x}(s)d(s) + \gamma_i^2\omega(t)^T\omega(t). \tag{5.17}$$

(5.17) can be written easily in following form

$$\left[\tilde{x}^T(t)\tilde{A}_i^T P_i + \tilde{x}^T(t-h)\tilde{A}_{hi}^T P_i + +f^T(t)\tilde{E}_i^T P_i + \tilde{\omega}^T(t)\tilde{B}_i^T P_i\right]\tilde{x}(t) + \tilde{x}^T(t)P_i\tilde{A}_i\tilde{x}(t)$$
$$+ \tilde{x}^T(t)P_i\tilde{A}_{hi}\tilde{x}(t-h) + \tilde{x}^T(t)P_i\tilde{E}_i f(t) + \tilde{x}^T(t)P_i\tilde{B}_i\tilde{\omega}(t) + \tilde{x}^T(t)e^{\lambda m}Q_i\tilde{x}(t)$$
$$- \tilde{x}^T(t-h)e^{\lambda m}Q_i\tilde{x}(t-h) + \tilde{x}^T(t)\tilde{C}_i^T\tilde{C}_i\tilde{x}(t) + \tilde{x}^T(t)\tilde{C}_i^T\tilde{C}_{hi}\tilde{x}(t-h) + \tilde{x}^T(t)\tilde{C}_i^T\tilde{F}_i f(t)$$
$$+ \tilde{x}^T(t)\tilde{C}_i^T\tilde{D}_i\tilde{\omega}(t) + \tilde{x}^T(t-h)\tilde{C}_{hi}^T\tilde{C}_i\tilde{x}(t) + \tilde{x}^T(t-h)\tilde{C}_{hi}^T\tilde{C}_{hi}\tilde{x}(t-h)$$
$$+ \tilde{x}^T(t-h)\tilde{C}_{hi}^T\tilde{F}_i f(t) + \tilde{x}^T(t-h)\tilde{C}_{hi}^T\tilde{D}_i\tilde{\omega}(t) + f^T(t)\tilde{F}_i^T\tilde{C}_i\tilde{x}(t) + f^T(t)\tilde{F}_i^T\tilde{C}_{hi}\tilde{x}(t-h)$$
$$+ f^T(t)\tilde{F}_i^T\tilde{F}_i f(t) + f^T(t)\tilde{F}_i^T\tilde{D}_i\tilde{\omega}(t) + \tilde{\omega}^T(t)\tilde{D}_i^T\tilde{C}_i\tilde{x}(t) + \tilde{\omega}^T(t)\tilde{D}_i^T\tilde{C}_{hi}\tilde{x}(t-h)$$
$$+ \tilde{\omega}^T(t)\tilde{D}_i^T\tilde{F}_i f(t) + \tilde{\omega}^T(t)\tilde{D}_i^T\tilde{D}_i\tilde{\omega}(t) + \lambda_m\tilde{x}^T(t)P_i x(t) + \lambda_m\int_{t-h}^{t}\tilde{x}^T(s)e^{\lambda m}Q_i\tilde{x}(s)d(s)$$
$$- {\gamma_i}^2\tilde{\omega}^T(t)\tilde{\omega}(t) \leq 0 \tag{5.18}$$

Further, the above inequality can be written as

$$\begin{bmatrix}\tilde{x}^T(t) & \tilde{x}^T(t-h) & f^T(t) & \tilde{\omega}^T(t)\end{bmatrix} M \begin{bmatrix}\tilde{x}(t) \\ \tilde{x}^T(t-h) \\ f^T(t) \\ \tilde{\omega}^T(t)\end{bmatrix} \leq 0 \;\; where, \tag{5.19}$$

$$M = \begin{bmatrix}\Delta_1 & P_i\tilde{A}_{hi} + {\tilde{C}_i}^T\tilde{C}_{hi} & P_i\tilde{E}_i + {\tilde{C}_i}^T\tilde{F}_i & P_i\tilde{B}_i + {\tilde{C}_i}^T\tilde{D}_i \\ * & -Q_i + \tilde{C}_{hi}^T\tilde{C}_{hi} & \tilde{C}_{hi}^T\tilde{F}_i & \tilde{C}_{hi}^T\tilde{D}_i \\ * & * & \tilde{F}_i^T\tilde{F}_i & \tilde{F}_i^T\tilde{D}_i \\ * & * & * & {\tilde{D}_i}^T\tilde{D}_i - \gamma_i^2 I\end{bmatrix}$$

where, $\Delta_1 = \tilde{A}_i^T P_i + P_i \tilde{A}_i + Q_i + \tilde{C}_i^T \tilde{C}_i + \lambda_m P_i$

for (5.19) to hold, it is required that

$$M < 0 \tag{5.20}$$

Applying Schur's compliment

$$\begin{bmatrix} \Delta_2 & P_i\tilde{A}_{hi} & P_i\tilde{E}_i & P_i\tilde{B}_i \\ * & -Q_i & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & -\gamma_i^2 I \end{bmatrix} + \begin{bmatrix} \tilde{C}_i^T\tilde{C}_i & \tilde{C}_i^T\tilde{C}_{hi} & \tilde{C}_i^T\tilde{F}_i & \tilde{C}_i^T\tilde{D}_i \\ * & \tilde{C}_{hi}^T\tilde{C}_{hi} & \tilde{C}_{hi}^T\tilde{F}_i & \tilde{C}_{hi}^T\tilde{D}_i \\ * & * & \tilde{F}_i^T\tilde{F}_i & \tilde{F}_i^T\tilde{D}_i \\ * & * & * & \tilde{D}_i^T\tilde{D}_i \end{bmatrix} \leq 0 \tag{5.21}$$

where, $\Delta_2 = \tilde{A}_i^T P_i + P_i \tilde{A}_i + Q_i + \lambda_m P_i$

further,

$$\begin{bmatrix} \Delta_2 & P_i\tilde{A}_{hi} & P_i\tilde{E}_i & P_i\tilde{B}_i \\ * & -Q_i & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & -\gamma_i^2 I \end{bmatrix} - \begin{bmatrix} \tilde{C}_i^T \\ \tilde{C}_{hi}^T \\ \tilde{F}_i^T \\ \tilde{D}_i \end{bmatrix} (-1)^{-1} \begin{bmatrix} \tilde{C}_i^T & \tilde{C}_{hi}^T & \tilde{F}_i^T & \tilde{D}_i \end{bmatrix} \leq 0 \tag{5.22}$$

using Schur's compliment property

$$\begin{bmatrix} \tilde{A}_i^T P_i + P_i\tilde{A}_i + Q_i + \lambda_m P_i & P_i\tilde{A}_{hi} & P_i\tilde{E}_i & P_i\tilde{B}_i & \tilde{C}_i^T \\ * & -Q_i & 0 & 0 & \tilde{C}_{hi}^T \\ * & * & 0 & 0 & \tilde{F}_i^T \\ * & * & * & -\gamma_i^2 I & \tilde{D}_i^T \\ \tilde{C}_i & \tilde{C}_{hi} & \tilde{F}_i & \tilde{D}_i & -I \end{bmatrix} \leq 0 \tag{5.23}$$

which is the basic form of LMI (5.11). Notice that, LMIs (5.7)-(5.10) are general requirements for model-based FD in asynchronous switching paradigm [93].

Unmatched Period

We consider augmented system (5.4), in this duration. Using Lemma 1,

$$\dot{V}_{ij}(\tilde{x}(t)) \leq -\lambda_u V_{ij}(\tilde{x}(t)) - r(t)^T r(t) + \gamma_{ij}^2 \omega(t)^T \omega(t) \tag{5.24}$$

Considering the following Lyapunov function,

$$V_{ij}(\tilde{x}(t)) = \tilde{x}^T(t)P_{ij}\tilde{x}(t) + \int_{t-h}^{t} \tilde{x}^T(s)e^{\lambda m}Q_{ij}\tilde{x}(s)d(s) \tag{5.25}$$

Differentiating (5.25) along the trajectory of (5.4)

$$\dot{V}_{ij}(\tilde{x}(t)) = \dot{\tilde{x}}^T(t)P_{ij}\tilde{x}(t) + \tilde{x}^T(t)P_{ij}\dot{\tilde{x}}(t) + \tilde{x}^T(t)e^{\lambda m}Q_{ij}\tilde{x}(t) - \tilde{x}^T(t-h)e^{\lambda m}Q_{ij}\tilde{x}(t-h) \tag{5.26}$$

after substituting (5.25)and (5.26), in (5.24),

$$\begin{aligned}
&\dot{\tilde{x}}^T(t)P_{ij}\tilde{x}(t) + \tilde{x}^T(t)P_{ij}\dot{\tilde{x}}(t) + \tilde{x}^T(t)e^{\lambda m}Q_{ij}\tilde{x}(t) - \tilde{x}^T(t-h)e^{\lambda m}Q_{ij}\tilde{x}(t-h) \\
&+ \left[\tilde{C}_{ij}\tilde{x}(t) + \tilde{C}_{hij}\tilde{x}(t-h) + \tilde{F}_{ij}f(t) + \tilde{D}_{ij}\tilde{\omega}(t)\right]^T \\
&\left[\tilde{C}_{ij}\tilde{x}(t) + \tilde{C}_{hij}\tilde{x}(t-h) + \tilde{F}_{ij}f(t) + \tilde{D}_{ij}\tilde{\omega}(t)\right] \\
&\leq -\lambda_m\tilde{x}^T(t)P_{ij}\tilde{x}(t) - \lambda_u\int_{t-h}^{t} \tilde{x}^T(s)e^{\lambda_u}Q_i\tilde{x}(s)d(s) + \gamma_{ij}^2\omega(t)^T\omega(t).
\end{aligned} \tag{5.27}$$

Substituting the expression for $\dot{\tilde{x}}(t)$ from (5.4) in (5.27),

$$\begin{aligned}
&\left[\tilde{A}_{ij}\tilde{x}(t) + \tilde{A}_{hij}\tilde{x}(t-h) + \tilde{E}_{ij}f(t) + \tilde{B}_{ij}\tilde{\omega}(t)\right]^T P_{ij}\tilde{x}(t) \\
&+ \tilde{x}^T(t)P_{ij}\left[\tilde{A}_{ij}\tilde{x}(t) + \tilde{A}_{hij}\tilde{x}(t-h) + \tilde{E}_{ij}f(t) + \tilde{B}_{ij}\tilde{\omega}(t)\right] \\
&+ \tilde{x}^T(t)e^{\lambda_u}Q_{ij}\tilde{x}(t) - \tilde{x}^T(t-h)e^{\lambda_u}Q_{ij}\tilde{x}(t-h) \\
&+ \left[\tilde{C}_{ij}\tilde{x}(t) + \tilde{C}_{hij}\tilde{x}(t-h) + \tilde{F}_{ij}f(t) + \tilde{D}_{ij}\tilde{\omega}(t)\right]^T \\
&\left[\tilde{C}_{ij}\tilde{x}(t) + \tilde{C}_{hij}\tilde{x}(t-h) + \tilde{F}_{ij}f(t) + \tilde{D}_{ij}\tilde{\omega}(t)\right] \\
&\leq -\lambda_u\tilde{x}^T(t)P_{ij}\tilde{x}(t) - \lambda_u\int_{t-h}^{t} \tilde{x}^T(s)e^{\lambda m}Q_{ij}\tilde{x}(s)d(s) + \gamma_{ij}^2\omega(t)^T\omega(t).
\end{aligned} \tag{5.28}$$

(5.28) can be written easily in following form

$$\begin{aligned}
&\left[\tilde{x}^T(t)\tilde{A}_{ij}^T P_{ij} + \tilde{x}^T(t-h)\tilde{A}_{hij}^T P_{ij} + +f^T(t)\tilde{E}_{ij}^T P_{ij} + \tilde{\omega}^T(t)\tilde{B}_{ij}^T P_{ij}\right]\tilde{x}(t)\\
&+\tilde{x}^T(t)P_{ij}\tilde{A}_{ij}\tilde{x}(t) + \tilde{x}^T(t)P_{ij}\tilde{A}_{hij}\tilde{x}(t-h) + \tilde{x}^T(t)P_{ij}\tilde{E}_{ij}f(t) + \tilde{x}^T(t)P_{ij}\tilde{B}_{ij}\tilde{\omega}(t)\\
&+\tilde{x}^T(t)e^{\lambda_u}Q_{ij}\tilde{x}(t) - \tilde{x}^T(t-h)e^{\lambda_u}Q_{ij}\tilde{x}(t-h) + \tilde{x}^T(t)\tilde{C}_{ij}^T\tilde{C}_{ij}\tilde{x}(t)\\
&+\tilde{x}^T(t)\tilde{C}_{ij}^T\tilde{C}_{hij}\tilde{x}(t-h) + \tilde{x}^T(t)\tilde{C}_{ij}^T\tilde{F}_{ij}f(t) + \tilde{x}^T(t)\tilde{C}_{ij}^T\tilde{D}_{ij}\tilde{\omega}(t) + \tilde{x}^T(t-h)\tilde{C}_{hij}^T\tilde{C}_{ij}\tilde{x}(t)\\
&+\tilde{x}^T(t-h)\tilde{C}_{hij}^T\tilde{C}_{hij}\tilde{x}(t-h) + \tilde{x}^T(t-h)\tilde{C}_{hij}^T\tilde{F}_{ij}f(t) + \tilde{x}^T(t-h)\tilde{C}_{hij}^T\tilde{D}_{ij}\tilde{\omega}(t)\\
&+f^T(t)\tilde{F}_{ij}^T\tilde{C}_{ij}\tilde{x}(t) + f^T(t)\tilde{F}_{ij}^T\tilde{C}_{hij}\tilde{x}(t-h) + f^T(t)\tilde{F}_{ij}^T\tilde{F}_{ij}f(t) + f^T(t)\tilde{F}_{ij}^T\tilde{D}_{ij}\tilde{\omega}(t)\\
&+\tilde{\omega}^T(t)\tilde{D}_{ij}^T\tilde{C}_{ij}\tilde{x}(t) + \tilde{\omega}^T(t)\tilde{D}_{ij}^T\tilde{C}_{hij}\tilde{x}(t-h) + \tilde{\omega}^T(t)\tilde{D}_{ij}^T\tilde{F}_{ij}f(t) + \tilde{\omega}^T(t)\tilde{D}_{ij}^T\tilde{D}_{ij}\tilde{\omega}(t)\\
&+\lambda_u\tilde{x}^T(t)P_{ij}x(t) + \lambda_u\int_{t-h}^{t}\tilde{x}^T(s)e^{\lambda_u}Q_{ij}\tilde{x}(s)d(s) - \gamma_{ij}{}^2\tilde{\omega}^T(t)\tilde{\omega}(t) \le 0
\end{aligned} \tag{5.29}$$

Further, the above inequality can be written as

$$\begin{bmatrix}\tilde{x}^T(t) & \tilde{x}^T(t-h) & f^T(t) & \tilde{\omega}^T(t)\end{bmatrix} M \begin{bmatrix}\tilde{x}(t)\\ \tilde{x}^T(t-h)\\ f^T(t)\\ \tilde{\omega}^T(t)\end{bmatrix} \le 0 \;\; where, \tag{5.30}$$

$$M = \begin{bmatrix}\Delta_2 & P_{ij}\tilde{A}_{hij} + \tilde{C_{ij}}^T\tilde{C}_{hij} & P_{ij}\tilde{E}_{ij} + \tilde{C_{ij}}^T\tilde{F}_{ij} & P_{ij}\tilde{B}_{ij} + \tilde{C_{ij}}^T\tilde{D}_{ij}\\ * & -Q_{ij} + \tilde{C}_{hij}^T\tilde{C}_{hij} & \tilde{C}_{hij}^T\tilde{F}_{ij} & \tilde{C}_{hij}^T\tilde{D}_{ij}\\ * & * & \tilde{F}_{ij}^T\tilde{F}_{ij} & \tilde{F}_{ij}^T\tilde{D}_{ij}\\ * & * & * & \tilde{D_{ij}}^T\tilde{D}_{ij} - \gamma_i^2 I\end{bmatrix}$$

where,

$\Delta_2 = \tilde{A_{ij}}^T P_{ij} + P_{ij}\tilde{A}_{ij} + Q_{ij} + \tilde{C_{ij}}^T\tilde{C}_{ij} + \lambda_u P_{ij}$

for (5.30) to hold, it is required that

$$M < 0 \tag{5.31}$$

Applying Schur's compliment

$$\begin{bmatrix} \Delta_2 & P_{ij}\tilde{A}_{hij} & P_{ij}\tilde{E}_{ij} & P_{ij}\tilde{B}_{ij} \\ * & -Q_{ij} & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & -\gamma_i^2 I \end{bmatrix} + \begin{bmatrix} \tilde{C_{ij}}^T\tilde{C}_{ij} & \tilde{C_{ij}}^T\tilde{C}_{hij} & \tilde{C_{ij}}^T\tilde{F}_{ij} & \tilde{C_{ij}}^T\tilde{D}_{ij} \\ * & \tilde{C}_{hij}^T\tilde{C}_{hij} & \tilde{C}_{hij}^T\tilde{F}_{ij} & \tilde{C}_{hij}^T\tilde{D}_{ij} \\ * & * & \tilde{F}_{ij}^T\tilde{F}_{ij} & \tilde{F}_{ij}^T\tilde{D}_{ij} \\ * & * & * & \tilde{D_{ij}}^T\tilde{D}_{ij} \end{bmatrix} \leq 0 \tag{5.32}$$

further,

$$\begin{bmatrix} \Delta_2 & P_{ij}\tilde{A}_{hij} & P_{ij}\tilde{E}_{ij} & P_{ij}\tilde{B}_{ij} \\ * & -Q_{ij} & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & -\gamma_i^2 I \end{bmatrix} - \begin{bmatrix} \tilde{C_{ij}}^T \\ \tilde{C}_{hij}^T \\ \tilde{F}_{ij}^T \\ \tilde{D}_{ij} \end{bmatrix} (-1)^{-1} \begin{bmatrix} \tilde{C_{ij}}^T & \tilde{C}_{hij}^T & \tilde{F}_{ij}^T & \tilde{D}_{ij} \end{bmatrix} \leq 0 \tag{5.33}$$

using Schur's compliment property

$$\begin{bmatrix} \Delta_2 & P_{ij}\tilde{A}_{hij} & P_{ij}\tilde{E}_{ij} & P_{ij}\tilde{B}_{ij} & \tilde{C}_{ij}^T \\ * & -Q_{ij} & 0 & 0 & \tilde{C}_{hij}^T \\ * & * & 0 & 0 & \tilde{F}_{ij}^T \\ * & * & * & -\gamma_{ij}^2 I & \tilde{D}_{ij}^T \\ \tilde{C}_{ij} & \tilde{C}_{hij} & \tilde{F}_{ij} & \tilde{D}_{ij} & -I \end{bmatrix} \leq 0 \tag{5.34}$$

which is the basic form of LMI (5.12). □

5.4 Threshold Computation and Residual Evaluation

In this work following residual evaluation function, which is based on RMS energy of the residual signal, is utilized, which has been discussed in the preceding Chapter 3.

$$J_{RMS} = \| r(t) \|_{RMS} = (\frac{1}{T}\int_t^{t+T} \| r(\tau) \|^2 \, d\tau)^{\frac{1}{2}} \tag{5.35}$$

where, T is the evaluation window.

Similarly, following threshold is employed, which is defined as,

$$J_{th,RMS,2} = \sup_{\|\omega(t)\|_2 \leq \delta_{d,2}+\delta_{u,2}, f(t)=0} J_{RMS}, \tag{5.36}$$

and computed as

$$J_{th,RMS,2} = \frac{\gamma_i^*(\delta_{d,2} + \delta_{u,2})}{\sqrt{T}} \tag{5.37}$$

where $\gamma_i^* = min(\gamma_i)$, $\delta_{d,2}$ denotes the set of L_2-norm bounded disturbance signals and $\delta_{u,2}$ denotes the set of L_2-norm bounded input signals. Finally, decision about the presence of fault in the system is made by the following logic

- $J_{RMS} \leq J_{th,RMS,2} \Longrightarrow$ No FAULT
- $J_{RMS} > J_{th,RMS,2} \Longrightarrow$ Detected FAULT

In the next subsection, we present the algorithm in stepwise simplified form to design our objective filter, which is based on the results derived in Theorem 5.

5.4.1 Algorithm

Let the model of the switched system is given as in 5.1. By taking into account the assumptions, in Chapter 2,

1. Check the detectability of all subsystems (modes), i.e., whether (A_i, C_i) is detectable $\forall i \in \{1, 2, ...N\}$, if yes then proceed to next step, if no, then it is not possible to proceed
2. Set the ADT parameters, μ_1, μ_2, α_i, and ρ_i, $\forall \quad i \in \{1, 2, ...N\}$ then solve the LMIs (5.7)-(5.12) simultaneously to get the optimal values of γ_i, and γ_{ij} $\forall \quad i \in \{1, 2, ...N\}$.
3. Find filter parameters L_i and $H_i \forall \quad for i \neq j$, and $i, j \in N$ from step 2, as well.

5.5 Application to the Case Study

In this section, proposed framework is utilized for fault detection in a switched time-delay system example. The Simulation platform is for time 30s. Subsystem 1 is activated when $\sigma(t) = 1$ and subsystem 2 when $\sigma(t) = 0$. Two faults, actuator fault ($f_1(t)$ = -2) and sensor fault ($f_2(t)$ = 2) are simulated. Details of switching behaviour of corresponding subsystems, filters and faults can be found in Fig.5.1. Switching signal $\sigma(t)$ is applied with ADT value of 1.6218, that is, switching interval between any two subsystems is greater than 1.6218. Further, for simulation study we take the disturbance signal as L_2 norm bounded with $\delta_{d,2} \leq 1$ for each mode. In practice noise signal is of stochastic nature, for simplification, here we assume the noise signal of deterministic nature for each mode.

5.5.1 A Numerical Example

A numerical example is studies to simulate results of proposed method. The considered switched time-delay system consisting of two modes, having state delays, is as follows:

$$A_1 = \begin{bmatrix} -1.1 & 0.1 \\ 1.1 & -1.3 \end{bmatrix}, B_1 = \begin{bmatrix} 2 \\ 0 \end{bmatrix} C_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, B_{d1} = \begin{bmatrix} 0.1 & 0.1 \\ 1.1 & 1.1 \end{bmatrix}, D_{d1} = \begin{bmatrix} 0.2 & 0.1 \\ 0.3 & 0.2 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -1.4 & 0.3 \\ 0.2 & -1.3 \end{bmatrix}, B_2 = \begin{bmatrix} 1.2 \\ 0 \end{bmatrix}, C_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, B_{d2} = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.1 \end{bmatrix}, D_{d2} = \begin{bmatrix} 0.2 & 0.1 \\ 0.2 & 0.1 \end{bmatrix},$$

$$A_{h1} = \begin{bmatrix} -0.2 & 0.1 \\ 0 & -0.1 \end{bmatrix}, A_{h2} = \begin{bmatrix} 0.02 & 0 \\ 0.1 & -0.1 \end{bmatrix}, C_{h1} = \begin{bmatrix} -0.2 & 0.1 \\ 0.1 & 0.1 \end{bmatrix}, C_{h2} = \begin{bmatrix} -0.1 & 0 \\ 0.2 & 0.1 \end{bmatrix}$$

$$B_{f1} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}, B_{f2} = \begin{bmatrix} 1.2 & 0 \\ 0 & 0 \end{bmatrix}, D_{f1} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, D_{f2} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

Results and Discussion

By using Theorem 5, solving LMIs (5.7)-(5.12), faults are simulated as in Fig.5.1. Under the above mentioned setting, The residual signals for the system in absence of faults is depicted in Fig. 5.2. It can be seen that the residual signal is not zero even when there is no fault present in the system. To this end, proper fault detection is not possible, and false alarms may be generated. That is the reason, residual signals need to be evaluated and then a threshold level has to be set for detecting the faults using (5.35) and (5.37). Evaluated residuals and thresholds have been plotted in figures Fig. (5.3) in case of no fault. In Fig.(5.4) evaluated residuals and thresholds are depicted for actuator fault while in Fig.(5.5) evaluated residuals and thresholds are shown in case of sensor fault. It is easy to see that the faults is detected effectively in very short span of time, when evaluated residual signal crosses the threshold level.

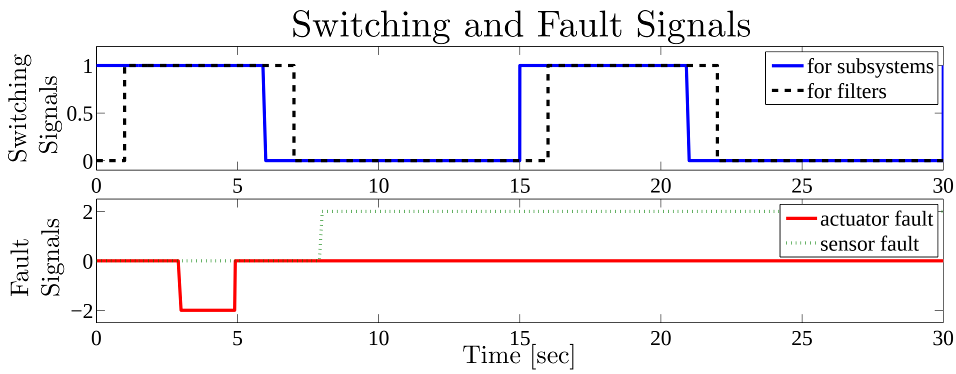

Figure 5.1: (a): Switching signals: for subsystems $\sigma(t)$; for filters $\sigma'(t)$ (b): Fault signals: of actuator fault $f_1(t)$; of sensor fault $f_2(t)$

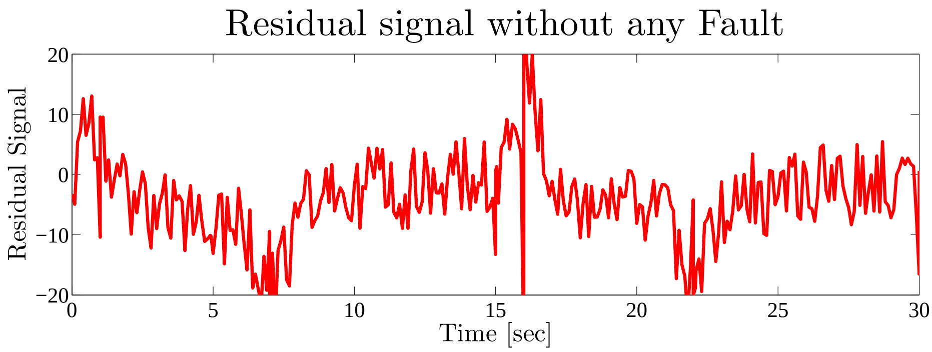

Figure 5.2: Residual , $r_1(t)$, without any fault

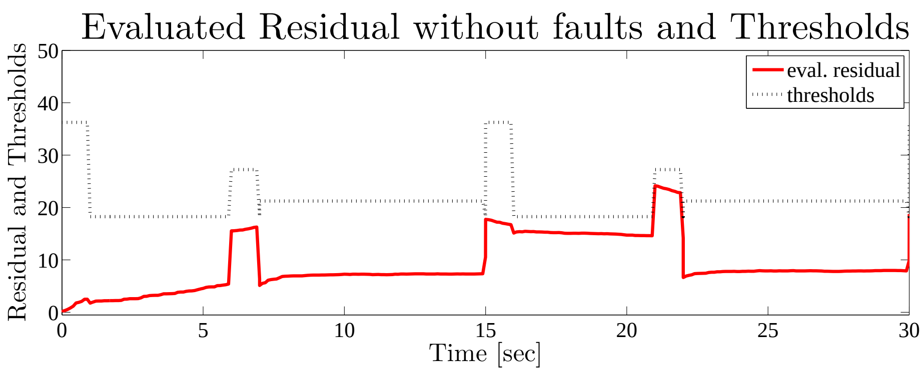

Figure 5.3: Evaluated residual without any fault, $er_1(t)$, and thresholds in both modes

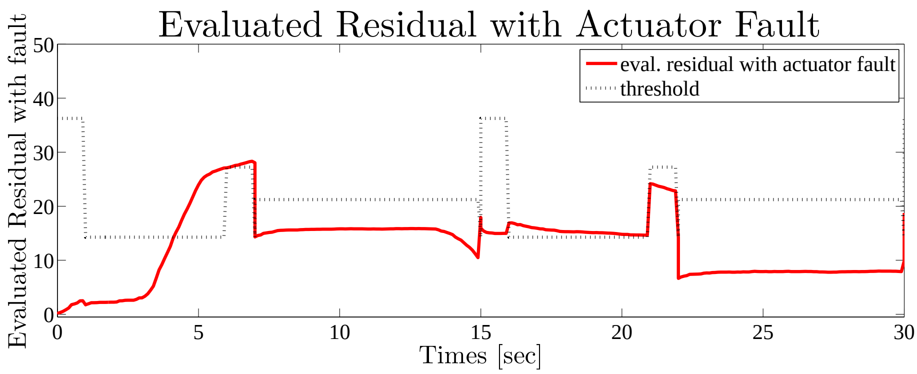

Figure 5.4: Evaluated residual when actuator fault occurring, $er_1(t)$, and thresholds

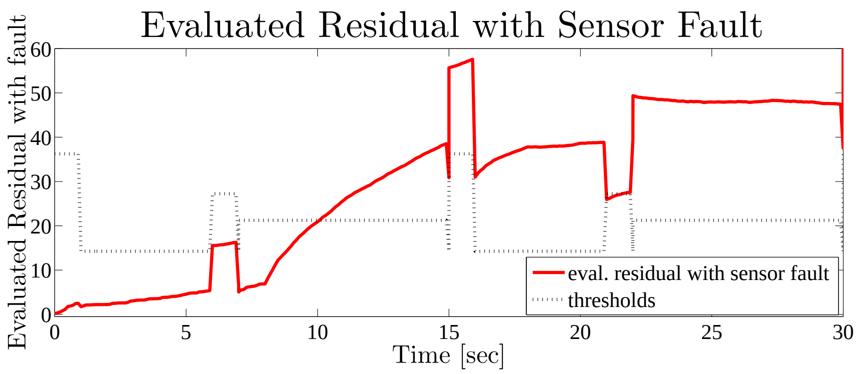

Figure 5.5: Evaluated residual when sensor fault occurring, $er_1(t)$, and thresholds

5.6 Summary

In this chapter, fault detection problem for switched time-delay systems is considered under asynchronous switching. FDF is designed as residual generator using H_∞ filtering approach. Sufficient conditions are proposed in terms of linear matrix inequalities. A numerical example is studied for the application of results. Actuator and sensor faults are successfully detected through proposed method.

Chapter 6

Conclusion and Future Recommendations

To improve the reliability and performance of switched systems, different strategies of fault diagnosis and fault tolerant control design have been proposed in the preceding chapters. This chapter presents the concluding remarks and indicates future directions for possible extension of this work.

6.1 Conclusion

This thesis is concerned with studying fault diagnosis and fault tolerant control design for switched systems under asynchronous switching. Novel approaches were proposed for fault diagnosis (FDD) and fault tolerant control (FTC) of switched systems. These approaches consist of; residual generation, residual evaluation, threshold computation, fault estimation, and fault tolerance. The prime objectives of this thesis were stated precisely in the first Chapter. To achieve these objectives, techniques were developed and proposed. The performance of these approaches was proven mathematically, and their effectiveness was illustrated by case studies. The features of switched systems and the importance of FDD/FTC in these systems was highlighted in Chapter 2. Fault detection (FD); and fault detection and isolation (FDI) schemes were investigated in Chapter 3. Fault estimation (FE) along with fault tolerant control (FTC) for switched systems was discussed in Chapter 4. Furthermore, the fault detection scheme for switched time-delay systems was presented in Chapter 5. The main problem in the focus has been the asynchronous switching paradigm in model-based FDD/FTC for switched systems. In this paradigm, the complete design chain of fault management system, i.e., from fault detection to fault tolerance, has been studied in the thesis.

6.2 Future Directions

Last section concluded the results obtained in this thesis. Besides the attainment of the proposed schemes, there is still room for further investigation. Some of these possible extensions and future directions are outlined below:

- In Chapter 3, schemes for fault detection and isolation were presented. The problem of FDI was considered with the assumption, that, number of outputs are greater than the number of faults to be isolated. In future, the methods can be explored for a less conservative technique.

- Proposed schemes in Chapters 3 successfully detect and isolate the faults in uncertain switched systems in asynchronous case, subject to norm bounded uncertainties. Proposed method can be applied easily to polytopic and stochastic type of uncertainties in switched systems. Another possible extension to Chapters 3 is to explore dynamic threshold setting at residual evaluation stage.

- Future recommendation related to Chapter 4, fault estimation and tolerance, is to develop adaptive fault estimation strategy. In addition, in case of complete failure of actuator or sensor, virtual actuator or virtual sensor approaches may be employed in the asynchronous switching paradigm. Furthermore, strategy may be developed for detecting time-varying faults.

- Fault detection for switched time-delay systems has been the focus of Chapter 5. Possible future directions of this research topic are to extend the features of fault isolation, estimation and tolerance for the faulty behaviour of switched time-delay systems.

- All the sample systems, which have been considered, are in continuous time. Recommendation for discrete time switched systems, to be investigated, is also one of the research area. The techniques for discrete-time switched systems may be a more attractive choice for implementation on embedded systems for industrial and practical systems.

- Practically nonlinear phenomenon occurs in systems. Similar to the preceding research path, area of nonlinear switched systems is also an interesting way to be traversed.

Appendices

Appendix

Lemmas and Theorems

Lemma 1. *(Schur's Compliment)[102]; According to Schur's compliment, the following two statements are equivalent*

1. $\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{12}^T & \Phi_{22} \end{bmatrix} < 0$
2. $\Phi_{22} < 0$, $\Phi_{11} - \Phi_{12}\Phi_{22}^{-1}\Phi_{12}^T < 0$

Lemma 2. *[5]; Let G, L, E and $F(t)$ are real matrices of appropriate dimensions with $F(t)$ being a matrix function and $F(t)^T F(t) \leq I$ then*
for any $\epsilon > 0$

$$LF(t)E + E^T F^T(t) L^T \leq \frac{1}{\epsilon} LL^T + \epsilon E^T E$$

Lemma 3. *[93] A switched system,*

$$\dot{x}(t) = A_i x(t) + B_i u(t)$$
$$y(t) = C_i x(t) + D_i u(t) \quad , i \in \{1, 2, ...N\}$$

is said to be globally asymptotically stable with average dwell time

$$\tau_a > \tau_a{}^* = \tfrac{ln(\mu 1 \mu 2)}{\zeta^*}, \tfrac{T^-(t_0,t)}{T^+(t_0,t)} \geq \tfrac{\rho+\zeta^*}{\alpha-\zeta^*}, 0 < \zeta^* < \alpha$$

and satisfies the H_∞ performance with index no greater than $\gamma = max(\gamma_i)$, if there exist Lyapunov functions $V_i(x(t))$ and $V_{ij}(x(t))$ $\forall i \in \{1, 2, ...N\}$ such that

$$
\begin{aligned}
&V_j(x(t)) \leq \mu_1 V_{ij}(x(t)) \\
&V_{ij}(x(t)) \leq \mu_2 V_i(x(t)) \\
&\dot{V}_i(x(t)) \leq -\alpha V_i(x(t)) - y^T(t)y(t) + {\gamma_i}^2 u^T(t)u(t) \\
&\dot{V}_{ij}(x(t)) \leq \rho V_{ij}(x(t)) - y^T(t)y(t) + {\gamma_{ij}}^2 u^T(t)u(t) \\
&\forall i, j \in \{1, 2, ...N\}, i \neq j.
\end{aligned}
$$

Notice that during $[t_o, t]$, $T^-(t_0, t)$ and $T^+(t_0, t)$ denote the total matched and mismatched periods, respectively. $T^+(t_0, t)$ is equal to Δ_i.

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