Kinematics Modeling of the Arm of a Radiation Protection Assistant Robot

Jahan Zeb, Nadeem Qaiser, Naeem Iqbal and Nasir Ahmed
Department of Electrical Engineering,
Pakistan Institute of Engineering and Applied Sciences, Islamabad Pakistan
jahan@pinstech.org.pk, nadeem.qaiser@pieas.edu.pk, nasir@pieas.edu.pk

Abstract

This paper presents the forward and inverse Kinematics modeling of a four Degree of Freedom (DOF) articulated robot arm, mounted on Radiation Protection Assistant Robot-1 (RPAR-I). It is a wheeled mobile monitoring and surveillance robot being designed and developed at Department of Electrical Engineering, Pakistan Institute of Engineering and Applied Sciences (PIEAS). Recent advances in robotics and radiation hardening technologies have led to the fusion of radiation protection techniques and robot devices. The robot equipped with machine vision and remote control capabilities for tool maneuvering, promises safety of radiation workers from unnecessary radiation doses during handling of radioactive material and preventive maintenance schedules.

Keywords: *Kinematics, articulated arm, Robotics, Radiation protection*

1. Introduction

Executing tasks hazardous to human beings are one of the main applications of robots besides performing automated and repeated works. That's why robots are becoming popular supporting tool in nuclear industry. Usually, profitability is the first motivation to switch from a regular worker to an automated system in a regular industry. However in nuclear industry, safety of the worker and the regulations are issues that cannot be ignored.

The national and international regulations on radiation protection indicate that a radiation worker should not receive dose more than 20 mSv per year [1]. It is quite expensive to meet the required dose limit. The workers dose cost more than $500,000 per man-Sv [2]. Introduction of robots in radiation work compensate this expense by reducing the manpower as much as that every dollar spent on robot is returned doubled [3]. Robotics can play an important role in helping to reduce the exposure to fulfill the ALARA (As Low As Reasonably Achievable) [4] concept.

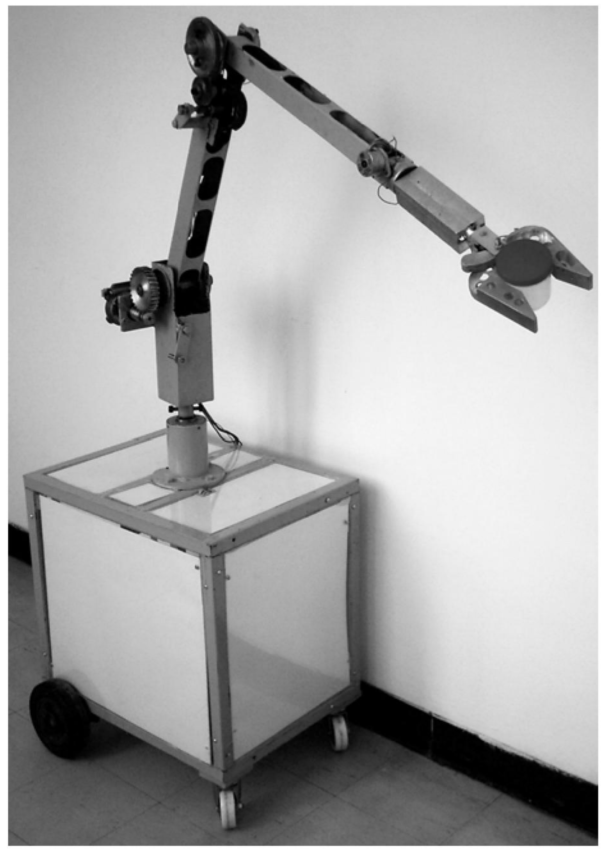

Figure 1. Radiation Protection Assistant Robot RPAR-I

In nuclear power plants, the reactor has to be shut down or at least brought to a fraction of its maximum power to allow human intervention near the core. Also, due to the aging of nuclear reactors, increased inspections and repairs are needed at a deeper level than ever before. Robots shorten the maintenance time and the number of workers needed; this reduction generates many additional savings: less protective clothes, waste and paperwork. Finally, an estimation of the hazardous environment in an emergency situation avoids putting personnel at risk. Furthermore, a remote system is often the only way to enter a very high radiation field.

The components to be used in the design of robots must be selected carefully. The performance of these components deteriorates with time in radiation field. In most cases, the robots are designed for "exceptional use," which means that they are operated only when they are needed, like in repair and inspection jobs. The components in the design of inspection and repair robots raise less shielding concerns, as they don't remain permanently in radiation field. In contrast, a robot whose aim is to manipulate radioactive equipment on a daily basis has a much greater need for radiation-tolerant components.

Demand of nuclear energy is increasing day by day due to rapidly depleting fossil fuel resources. For instance Pakistan plans to double its nuclear-based power generation in coming 10 years. This need also causes the repeated radiation exposure of workers in countries like Pakistan where robots are not currently in use in nuclear power plants.

RPAR-I is a robot developed for the said purpose. It is a mobile platform having 4 DOF articulated robot arm equipped with tele-operation and computer vision. This provides better vision and planning capabilities to the remote operator. In tele-operation mode, wipe test of radiation sources or handling of fresh radioactive sample from a reactor is safe as it avoids unnecessary radiation exposure.

This paper presents the kinematics modeling of robotic arm of RPAR-I The paper has been organized as follows. Section 2 describes briefly the design of RPAR-I, section 3 discusses the forward and inverse kinematics of the robot. Section 4 explains Jacobian of the manipulator, Section 5 illustrates the workspace of RPAR-I, followed by conclusion and future work plan in section 6.

2. Design of RPAR-I arm

A schematic of the robotic arm mounted on RPAR-I is shown in Figure 2. The four joints are (1) waist, (2) shoulder, (3) forearm and (4) wrist, having. net articulation range of 360°, 109°, 200° and 360° respectively. Joint i rotation (θ_i), occurs about the

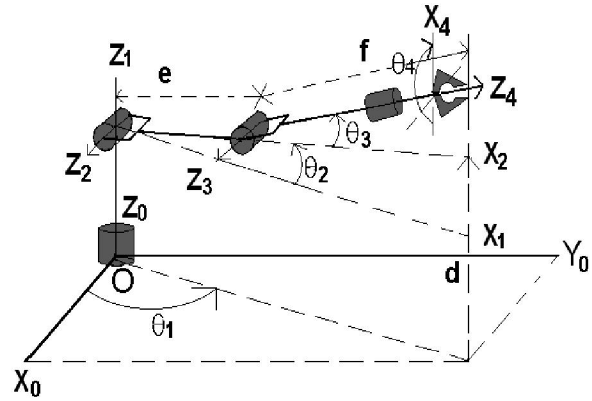

Figure 2. Geometry of robotic arm

$z_i - axis$, and all of the link frames are centered at point O but are shown at offsets for clarity. The End-effector is a gripper similar in shape to ERI Gripper arm [5]. However its dimensions, thickness, material and control are specially designed for secure handling of radioactive material. The gripper allows RPAR-I to grasp, transport and release cylindrical, flat and cubical objects.

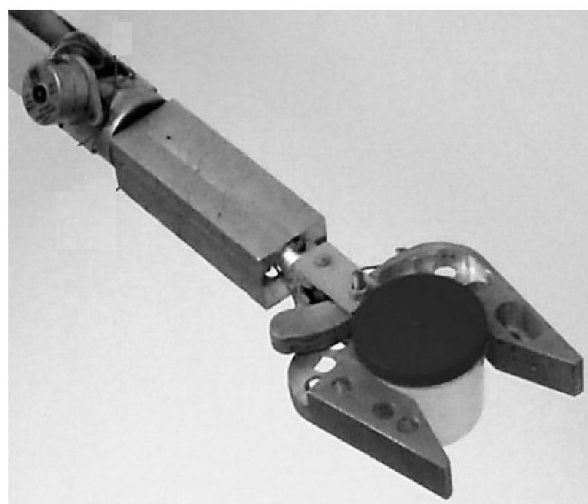

Figure 3. Gripper of RPAR-I

It can grasp any object that is smaller than $3''$ x $3.5''$ at the point being gripped. The weight of gripper is 1.75 lbs and it can hold objects up to 2.5 lbs without disturbing platform stability.

Power electronics to drive joints, end-effector and transportation support is housed in the mobile platform. Which also contains the on-board embedded computer, communication channel, vision modules, Info-lithium battery with charging circuitry.

3. Kinematics

Kinematics studies the geometric properties of the motion of fixed points in links without regard to their masses or the forces acting on them. This section presents detailed derivation of both forward and inverse kinematics of RPAR-I.

3.1. Forward kinematics

The forward kinematics problem in robotics is concerned with the transformation of position and orientation information in a joint space to a Cartesian space described by a forward kinematics equation

$$r(t) = f(\theta(t)) \tag{1}$$

Where

$\theta(t)$ is m-vector of joint variables

$r(t)$ is n-vector of Cartesian variables;

$f(\cdot)$ Continuous non-linear function whose structure and parameters are known for a given manipulator [6].

The forward kinematics of RPAR-I are derived by assigning link frames according to the *Denavit-Hartenberg (DH) Convention* [7, 8]. In D-H Notation, z_i is along the i^{th} joint axis, and θ_i is the angle between x_{i-1} and x_i about the $z_i - axis$. The out-of plane axes can be obtained by using the right-handed rule on each pair of link axes. The generalized rotation matrix, ${}^{j}R_i$, is the direction cosine matrix relating link frame i to link frame j.

$$^{j}R_i = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & b_i \\ \cos\alpha_1 \sin\theta_i & \cos\alpha_1 \cos\theta_i & -\sin\alpha_1 & -d_i \sin\alpha_1 \\ \sin\alpha_1 \sin\theta_i & \sin\alpha_1 \cos\theta_i & \cos\alpha_1 & d_i \cos\alpha \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

The D-H parameters for the robotic arm of RPAR-I are listed in Table 1[9].

Table 1. D-H parameters for the robotic arm

DH Parameters \ Link	1	2	3	4
α_i	0^0	90^0	0^0	90^0
b_i	0	0	e	f
θ_i	θ_1	θ_2	θ_3	θ_4
di	h	0	0	0

For RPAR-I: e =45.72, f = 76.2 and h =40.64 cm.

After putting the D-H parameters for the robotic arm of the RPAR-I in the generalized rotation matrix, ${}^{j}R_i$, the rotation matrix of positions and Orientation of each link are as follows

$$^{1}R_0 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & h \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

$$^{2}R_1 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

$$^{3}R_2 = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & e \\ \sin\theta_3 & \cos\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

$$^{4}R_3 = \begin{bmatrix} \cos\theta_4 & -\sin\theta_4 & 0 & f \\ 0 & 0 & -1 & 0 \\ \sin\theta_4 & \cos\theta_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

The resulting expression for the orientation of the end-effector with respect to the forearm, ${}^{0}R_4$ is found as following [10, 11] and a lengthy expression given in the appendix.

$$^{4}R_0 = {}^{1}R_0\, {}^{2}R_1\, {}^{3}R_2\, {}^{4}R_3 \tag{7}$$

The position equation of the end effector can be expressed as

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \begin{bmatrix} f\cos\theta_1 \cos\theta_{23} + e\cos\theta_1 \cos\theta_2 \\ f\sin\theta_1 \cos\theta_{23} + e\sin\theta_1 \cos\theta_2 \\ f\sin\theta_{23} + e\sin\theta_2 + h \end{bmatrix} \tag{8}$$

3.2. Inverse kinematics

The inverse kinematics problem is to find the joint variables given the desired positions and orientations of the end-effector through the inverse mapping of the forward kinematics [12].

$$\theta(t) = f^{-1}(r(t)) \tag{9}$$

The inverse kinematics problem involves the existence and uniqueness of a solution, and effectiveness and efficiency of solution methods. The inverse kinematics problem is thus much more difficult to solve than the forward kinematics problem for serial-link manipulators. The difficulties are compounded by the requirement of real-time solutions in sensor-based robotic operations. Therefore, real-time solution procedures to the inverse kinematics problem of redundant manipulators are of importance in robotics.

The inverse kinematics problem for RPAR-I is determining the particular values of the waist rotation angle θ_1, the shoulder rotation angle θ_2 and the elbow rotation angle θ_3, which will produce a known and desired gripper or tool origin position given as

$$(D^4_{x0}, D^4_{y0}, D^4_{z0}) = (P_x, P_y, P_z)$$

The positional equations for RPAR-I are derived and are given in eqs (10-16). As explicit from equations these are nontrivial and unrepeated positional relations.

$$f \cos\theta_1 \cos\theta_{23} + e\cos\theta_1 \cos\theta_2 = P_x \quad (10)$$

$$f \sin\theta_1 \cos\theta_{23} + e\sin\theta_1 \cos\theta_2 = P_y \quad (11)$$

$$f \sin\theta_{23} + e\sin\theta_2 + h = P_z \quad (12)$$

$$e\cos\theta_2 + f\cos\theta_{23} = \cos\theta_1 P_x + \sin\theta_1 P_y \quad (13)$$

$$\cos\theta_1 P_y - \sin\theta_1 P_x = 0 \quad (14)$$

$$e + f\cos\theta_3 = \cos\theta_2(\cos\theta_1 P_x + \sin\theta_1 P_y) + (P_z - h)\sin\theta_2 \quad (15)$$

$$f\sin\theta_3 = -\sin\theta_2(\cos\theta_1 P_x + \sin\theta_1 P_y) + (P_z - h)\cos\theta_2 \quad (16)$$

Solving the above positional equations for θ_1, θ_2 and θ_3

$$\theta_1 = A\tan 2\left(\frac{P_y}{P_x}\right) \quad (17)$$

$$\theta_2 = A\tan 2\left(\frac{(P_z - h)(e + f\cos\theta_3) - f\sin\theta_3\sqrt{P_X{}^2 + P_Y{}^2}}{(P_z - h)f\sin\theta_3 + (e + f\cos\theta_3)\sqrt{P_X{}^2 + P_Y{}^2}}\right) \quad (18)$$

$$\theta_3 = A\tan 2\left(\frac{\pm\sqrt{4e^2 f^2 - [(P_z - h)^2 + P_x{}^2 + P_y{}^2 - e^2 - f^2]^2}}{(P_z - h)^2 + P_x^2 + P_y{}^2 - e^2 - f^2}\right) \quad (19)$$

4. Manipulator Jacobian

The manipulator Jacobian represents the infinitesimal relationship between the joint displacements and the end-effector location at the present position and arm configuration [13]. The Jacobian matrix of the four degree-of-freedom manipulator is given in the appendix.

The elements of the Jacobian are functions of joint displacements, and therefore vary with the arm configuration.

Consider the instant when the three joints of RPAR-I move at joint velocities $\dot{\theta} = [\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3]^T$, and let $v = [\dot{x}, \dot{y}, \dot{z}]^T$ be the resulting end-effector velocity vector. The Jacobian represents the relationship between the joint velocities and the resulting end-effector velocities as well as the infinitesimal position relationship. Dividing both sides of Eq (8) by the infinitesimal time dt, we obtain

$$\frac{dDx}{dt} = j\frac{d\theta}{dt}$$

that is,

$$V = J\dot{\theta} \quad (20)$$

Thus the manipulator Jacobian determines the velocity relationship between the joints and the end-effector. Let J_1, J_2 and J_3 be three $3 \ x \ 1$ vectors consisting respectively of the first, second and third columns of the Jacobian. So, equation (20) can be written as

$$V = J_1\dot{\theta}_1 + J_2\dot{\theta}_2 + J_2\dot{\theta}_3 \quad (21)$$

The first term on the right-hand side accounts for the end-effector velocity induced by the first joint only. The second and third terms represent the velocities resulting from second and third joint respectively. The resultant end-effector velocity is given by the vector sum of the three. Each column vector of the Jacobian matrix represents the end-effector velocity generated by the corresponding joint motion at unit velocity when all other joints are immobilized.

5. Workspace analysis

The reachable workspace for RPAR-I is depicted as in figures 4 to 6. It is plotted by using position equations of the gripper (8), for different articulation ranges of θ_1, θ_2, and θ_3.

For these figures the variation of angles include $\theta_1 = 0^0$ to 360^0, $\theta_2 = -19^\circ$ to 90°. Different partial workspaces have been obtained for different values of θ_3. Figure 4 shows the maximum surface accessible to the gripper, of course limited by an internal minimum surface not shown.

It is plotted for $\theta_3 = 0^0$ i.e. fully extending the elbow. Figure 5. shows the workspace for $\theta_1 = 0^0$ to 270^0, $\theta_2 = -19°$ to $90°$ and for $\theta_3 = 0^0$. The range of θ_1 has been restricted to expose the bottom circle line of accessible space. Figure 6. Shows the minimum workspace at which the gripper can reach. It is plotted for $\theta_1 = 0^0$ to 360^0, $\theta_2 = 90°$ and for $\theta_3 = 0^0$ to 180^0.

6. Conclusions

RPAR-I has been designed and developed. Forward and inverse kinematics are discussed and calculated in detail. The manipulator Jacobian matrix has been found to transform velocities in joint space to velocities of the end-effector of robotic arm mounted on the RPAR-I in Cartesian space. Workspace and range of the robotic arm is estimated by considering the mobile as a stationary platform.

Future work plan includes estimation of essential robot parameters like center of masses and inertia tensors for the joints. By including these parameters a dynamical model of the robot for controller design purpose will be created using the Euler Langrangian Model.

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Appendix

Expression for Matrix $^0R_4$

In the following, c_i denotes $\cos(\theta_i)$ and s_i denotes $\sin(\theta_i)$. The wrist forward kinematics and Jacobian are all derived with respect to forearm frame:

$$^4R_0 = \begin{bmatrix} c_1c_4c_{23}+s_1s_4 & -c_1s_4c_{23}+s_1c_4 & c_1s_{23} & fc_1c_{23}+ec_1c_2 \\ s_1c_4c_{23}-c_1s_4 & -s_1s_4c_{23}-c_1c_4 & s_1s_{23} & fs_1c_{23}+es_1c_2 \\ c_4s_{23} & -s_4s_{23} & -c_{23} & fs_{23}+es_2+h \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

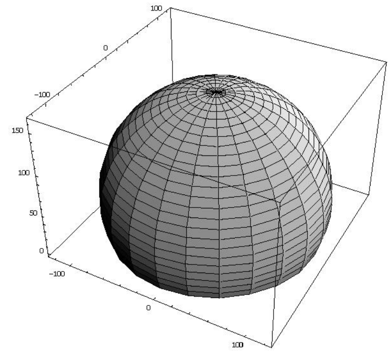

Figure 4. Workspace diagram of RPAR-I

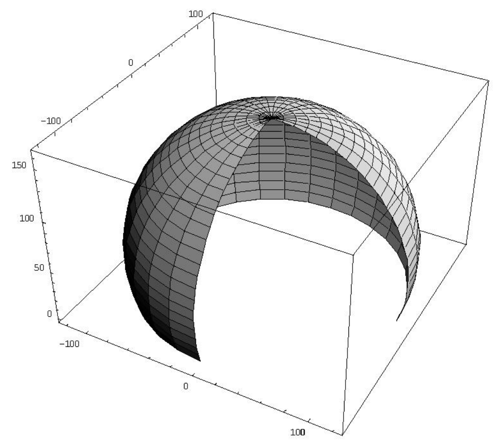

Figure 5. Partial Workspace diagram of RPAR-I

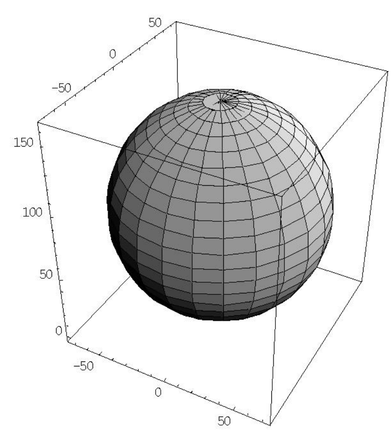

Figure 6. Minimum Workspace diagram of RPAR-I

Expression for the Jacobian Matrix for the end-effector

$$
{}^{0}J_{w} = \begin{bmatrix} -s_1(ec_2 + fc_{23}) & -c_1(es_2 + fs_{23}) & -fc_1 s_{23} \\ c_1(ec_2 + fc_{23}) & -s_1(es_2 + fs_{23}) & -fs_1 s_{23} \\ 0 & fc_{23} + c_2 e & fc_{23} \\ 0 & s_1 & s_1 \\ 0 & -c_1 & -c_1 \\ 1 & 0 & 0 \end{bmatrix}
$$

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